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BINOMIAL THEOREM

ALGEBRA

PRIME NUMBERS

DISTRIBUTIONS

IMAGINARY NUMBERS

INEQUALITIES

GEOMETRY

DIFFERENTIATION

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BINOMIAL THEOREM

INTEGRATION

LOGARITHMS

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QUADRATIC EQUATIONS

SEQUENCES

TRIGONOMETRY

Introduction

This book was written to help answer one of the common questions heard in maths classrooms up and down the country, and even once in the Houses of Parliament – what's the point of learning all this maths stuff?

Sometimes when you're working hard on the detail, learning the steps for doing maths, you can lose sight of the bigger picture, the 'why' part of it all.

We can help. Each of these sheets deals with an area of maths. All your 'favourites' are here, algebra, trigonometry, statistics and so on, showing you why they are useful and what you can do with them.

Maths lets us understand and describe the world in a really useful way, predicting how the complex elements of life interact, and helping build new and useful technologies.

Google works because of maths, the once theoretical mathematics of the search engine has made billions; million-selling video games are maths telling a good story; today's medicine uses maths. Almost every job in the world requires some maths to work.

Working with maths also helps you see the world in a different way. The logic and organisation of a mathematical brain can give you ways to solve problems and give you the edge in the job market, but it also opens up whole new ways to be creative, and yes even to have fun and enjoy maths. Hopefully after you've read this booklet, you'll get the point.

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Ever got lost following sat nav directions? Let's say you are lost in a town you have never been in before. You have to get to a friend's house and there are no street signs. Avraham Trahtman solved the Road Colouring Problem that could really help. The directions will work no matter what.

Town planners use maths as they design the functionality of towns and cities, particularly the shaping and uses of safe public spaces, recreational facilities, business and transport needs. They also use mathematical models to forecast the future needs of groups of people such as new stadia for football clubs or retail parks and out-of-town supermarkets.

Farming was one of the earliest applications of maths. There are records from ancient Egypt in 1850 BC that show maths being used to predict when the Nile would flood.

It is possible to build a bridge by trial and error and hope it doesn't collapse. However, modern engineering uses maths to design light-weight structures that we know will not fall over.

Flooding can cause severe damage and cause financial havoc for residents, businesses, insurance companies and governments alike. Simple and cost-effective flood defences can be made using embankments made of earth placed at strategic points. Simple volume calculations (and a bit of numerical integration) can help estimate the amount of earth required to prevent water from streams and rivers getting into homes.

Is the ground flat or not? Cartographers once had to measure hills and use trigonometry, but now satellites can look down and use maths to provide a three-dimensional model of the whole Earth.

what's the point of...
THE BINOMIAL THEOREM?

The binomial theorem and taste testing

When you learn algebra at school, it's not long before your teacher starts making you 'expand brackets'. You then never escape from these expanding brackets and one day you may find yourself learning about the binomial theorem. But why would anyone need to know about the binomial theorem?

The binomial theorem is a short cut so that you don't have to expand brackets and then simplify the terms. Mathematicians from Euclid in 400 BC onwards have noticed this short cut. It was eventually formalised by Blaise Pascal in a pamphlet that was published in 1665, shortly after he died.

If you are surveying people to find out which cola drink they prefer you could get lots of people to choose between the two products and see which one gets picked the most. If cola A is selected more than cola B, you could conclude that cola A tastes better. But what if they actually taste equally good? It could have been that cola A was selected more often just by random chance.

If the two drinks actually taste equally good, then they each have a 50% chance of being selected. So what if four, or even five, out of five people selected cola A?

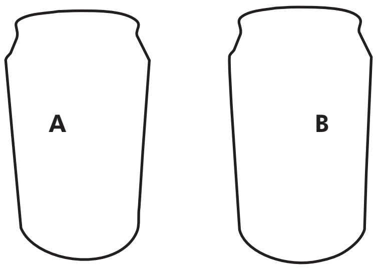
Is this enough to mean that it's better or could this be random chance?

If you let a stand for the probability that someone prefers cola A and b for the probability that someone prefers cola B, then you can represent five people making a choice like this:

$$(a + b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5a\, b^4 + b^5$$

For the situation where cola A and cola B have the same probability of being chosen, ($a = b = 0.5$) you get the following values:

Number of people choosing cola A	Number of people choosing cola B	Probability
5	0	0.031 25
4	1	0.156 25
3	2	0.312 50
2	3	0.312 50
1	4	0.156 25
0	5	0.031 25



So this means that the probability of four or five out of five people choosing cola A is 0.1875 or 18.75% when the colas taste equally good. You should be very suspicious of the conclusion that cola A definitely tastes better.

Relativity

Part of the power of the binomial theorem is its ability to speed up the use of complicated equations. In the equations that govern Einstein's theory of relativity there is this term.

$$\frac{1}{\sqrt{1-x}}$$

We can use the binomial theorem to expand this.

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots$$

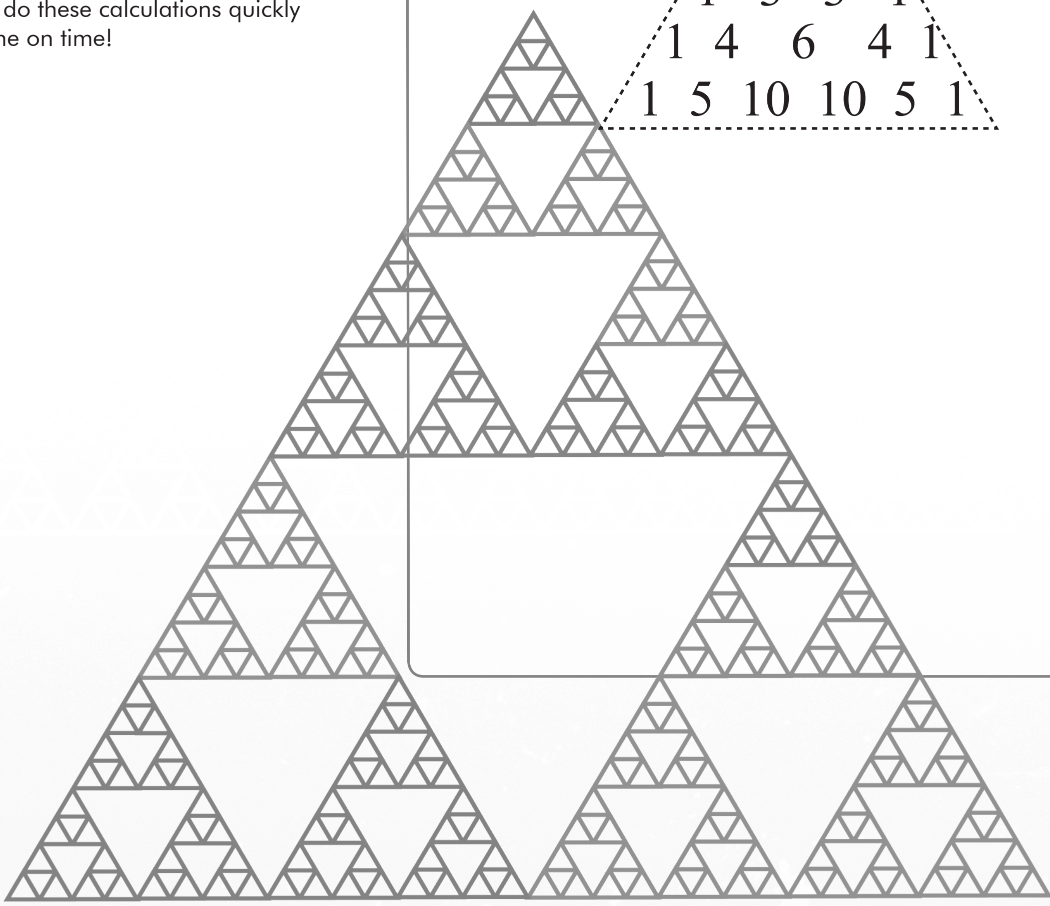
When a satnav uses GPS coordinates to calculate its position, the GPS satellites are moving in a different gravitational field to your car and so the calculations need to allow for relativistic time dilation, where time appears to move at different speeds when observed from different moving objects. Thanks to the binomial theorem, a satnav can do these calculations quickly enough to get you home on time!

Making pictures

If you write the coefficients of the binomial theorem in a triangle then you end up with Pascal's triangle, where each number equals the sum of the two above.

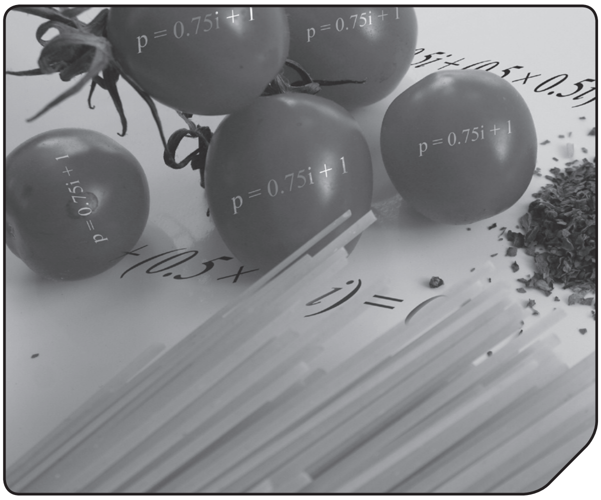
Then, if you colour in all of the odd numbers, you end up with the fractal known as the Sierpinski triangle. The Sierpinski triangle is created by taking a triangle, splitting it into four equal triangles and removing the middle one, then continually repeating this process on the new triangles created.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1



what's the point of... ALGEBRA?

How many make a crowd?



If $2x + 1 = 7$ what is x? To solve for x, subtract 1 from each side, so $2x = 6$, so $x = 3$, so what! Algebra, what's the point?

Algebra is used all over the place: if we aren't sure what a value can be, for example the number of people who are going to be at my party, we can use a letter to represent that value. Let's call the total number of people at my party p, where that p could be anything from a popular 70 to a sad 2.

The number of people will depend on the number of people I invite, let's call that number i, and suppose we estimate that only half of them will actually turn up, that will give us $0.5i$ people. Of those that do turn up, about half will bring someone else with them, so that's an extra $0.5 \times (0.5 \times i)$ people. So the total number of people will be $p = 0.5i + (0.5 \times 0.5i) = 0.75i$, oh and I'll be there too so that's $p = 0.75i + 1$.

Of course they'll all want something to eat. The recipe for my famous spag boll says I'll need 1 kg of spaghetti for five people, but how much will I need for the party? The amount of spaghetti needed per person is 0.2 kg, so for the party I'll need $s = 0.2 \times (0.75i + 1)$ kg, so that everyone can taste the wonder! I now, thanks to algebra, have a way of predicting how much spaghetti, or any other party ingredient, I need based on the number of invitations so I can estimate the cost of the party. So now it's decision time, how many invitations?

Solving almost any problem in life which involves money, time, distance, the amount or size of something or even simply comparing prices when shopping, all use algebra.

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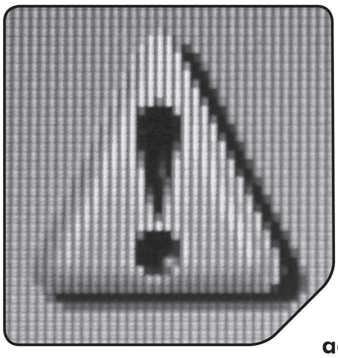


Zeroing in on a puzzle with algebra

Historically the word 'algebra' is believed to come from the Arabic word 'al-jabr', which scholars believe refers to the act of restoration or balancing.

In algebra the equations you have will contain an equals sign and the rule is what you do to one side of the equation, you do to the other side too, to keep the equation balanced. But you need to be careful with algebra. The steps below all seem to follow one after another but something must have gone wrong somewhere along the line. Can you spot the mistake in this algebraic balancing puzzle?

$a = x$	(which is true for some a's and x's)
$a^2 = ax$	(multiply both sides by a)
$a^2 - x^2 = ax - x^2$	(subtract x^2 from both sides)
$(a + x)(a - x) = x(a - x)$	(factorise)
$a + x = x$	(divide both sides by $a - x$)
$2x = x$	(as $a = x$)
$2 = 1$	(divide both sides by x)

Two is equal to one – I don't think so!
Where did it go wrong?

The mistake is near the end, when we did the division. We said at the start that a equals x, so $(a - x)$ is zero and dividing by zero just doesn't work mathematically. Think about it. How many nothings are there in something?

Many mathematical theories have come to an abrupt end because of this divide by zero problem. You can't do it; if you try you get nonsense!

A well-known example of why this sort of algebraic error is important in the real world is the case of the US navy ship *Yorktown*. In its time it was state-of-the-art computer controlled but, in September 1997 while on manoeuvres, a crew member entered a zero by mistake into the ship's software. The computer system couldn't cope with this error and the computer control systems failed leaving the ship without any working engines for a few hours.

Using algebra to check that computer software is correct is now big business. In safety-critical computer systems, such as deploying aeroplane landing gear, algebra plays a key role in checking that all the possible inputs lead to safe outputs.

A drive for using algebra

Algebra isn't just about solving textbook equations like $2x + 1 = 7$; it's a way to mathematically model the world. The word model here doesn't have anything to do with catwalks, it's a scientific term that refers to a way of building a description of sometimes complicated situations using letters or symbols to represent possible values. $v = u + at$ is a mathematical model: it tells you how fast your car will be going if it starts at speed u and accelerates for t seconds with acceleration a. Slot in the values you fancy and, vrooom, you have the answer you need. Car designers, in fact designers of all types of products, and also architects and engineers, use algebra all the time to ensure that they know exactly how the things they are making will turn out. Algebra is even part of the process for designing and testing new medicines. Without these mathematical models and the power of algebra to capture the way the world works we wouldn't have the products we all need to get along. Salesmen and truck drivers use algebra to calculate mileage, nurses use it to give correct doses of medicines and many others need to do some to fill in that dreaded tax return.

what's the point of...

PRIME NUMBERS?

Maths makes the world go round

Every time money is moved over the internet, whether it's multi-million pound transactions or payments of less than a pound, prime numbers are used to make sure that the money is moved securely.

When data, such as your bank card details, is sent over the internet it needs to be sent in code, or encrypted, so that if it was intercepted by criminals they wouldn't be able to use it. Encrypting data uses a 'key'. Early methods used the same key to both encrypt and decrypt messages. The problem with sending data over the internet is that the key for encrypting data needs to be freely available so that anyone can send in a transaction (this is known as the 'public key') but the key for undoing the code needs to be secret so that if criminals intercept the message they can't decrypt it (this is known as the 'private key').

It is very difficult to find a system where you can't easily find the private key if you know the public one. In the 1970s, however, three American mathematicians, Ron Rivest, Adi Shamir and Leonard Adleman developed a method based on prime numbers. This method has been named RSA after them.

RSA uses the idea that multiplying two large prime numbers together is relatively easy but factorising them is much more difficult.

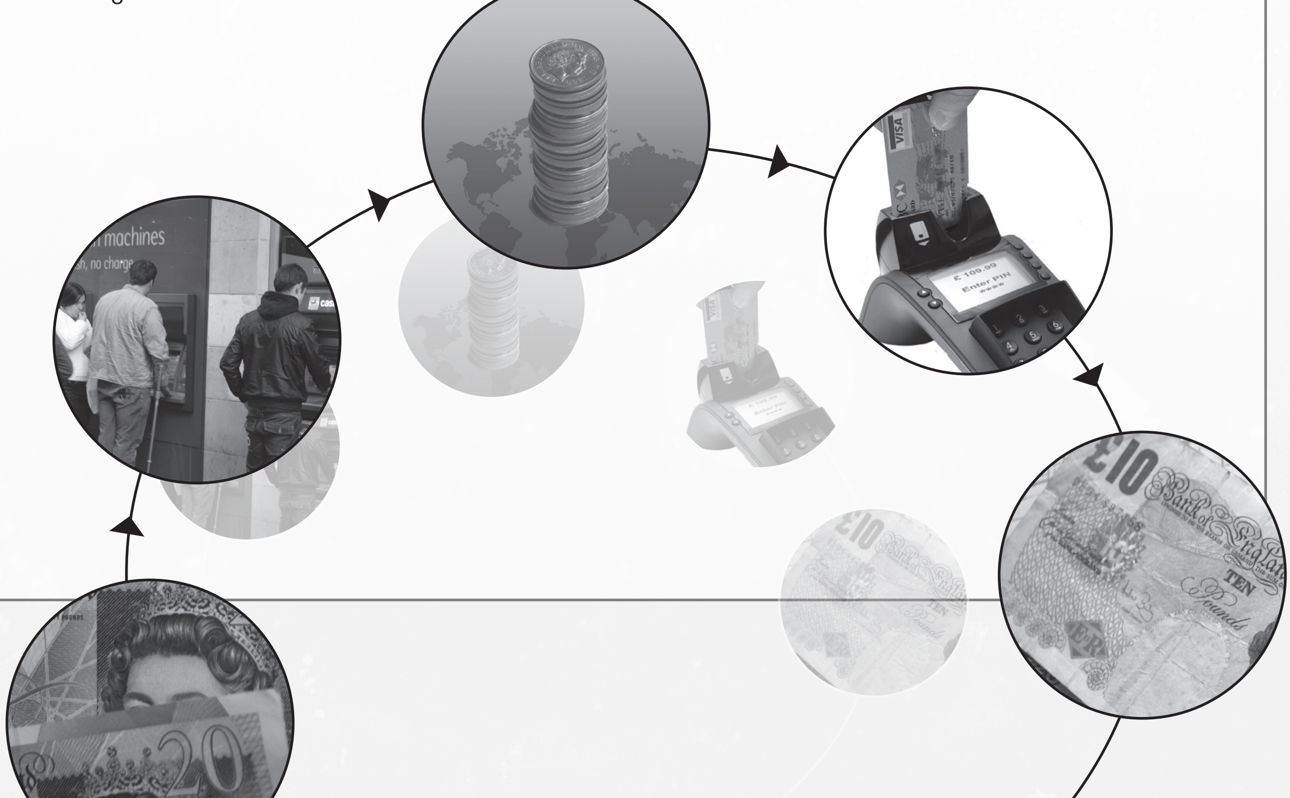
The public key uses the product of two large prime numbers and the private key uses the two prime numbers separately. If you think it sounds easy, try to find which two 8-digit prime numbers have been multiplied together to give

1 427 462 380 339 871.

Secure systems over the internet use prime numbers with a hundred digits or more!

If you use a bank card to buy something from a website your card details are encrypted and sent over the internet. If someone intercepts this encrypted message it will be meaningless – only the bank, with its knowledge of the private key, will be able to decrypt and find out your card number. This security isn't just restricted to financial transactions, for example a similar method can also be used when sending emails: you can digitally 'sign' emails to prove they came from you.

In the early part of the 20th century the mathematician G H Hardy worked on prime numbers. He was fiercely proud of being a Pure Mathematician and famously stated "Nothing I have ever done is of the slightest practical use". Prime numbers are now at the heart of countless secure transactions every day!



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Maths is useful in my book!

Books have a unique number on them to identify them. Before 2005 a 10-digit number was used, known as the International Standard Book Number (ISBN).

The first nine numbers identify the book and the last is a check digit. The last digit is generated by multiplying the first digit by 10, the second by 9 and so on, then adding up all the results. The remainder when this is divided by 11 is subtracted from 11 to give the check digit. (The remainder could be 10, in which case an X is used, or the number may be exactly divisible by 11, in which case a 0 is used.)

For example 0951611208 is a 10-digit ISBN number.
$0 \times 10 + 9 \times 9 + 5 \times 8 + 1 \times 7 + 6 \times 6 + 1 \times 5 + 1 \times 4 + 2 \times 3 + 0 \times 2 = 179$
This gives a remainder of 3 when divided by 11.
$11 - 3 = 8$ so it is likely that the ISBN number is correct.

Because 11 is a prime number it doesn't have any factors in common with the multipliers (10, 9, ..., 2). If a mistake has been made by copying down a number incorrectly or confusing the order of two of the numbers then the check digit will be incorrect.

Since 2005 13-digit ISBNs have been used but finding a suitable number, large enough to not have factors in common with the multipliers and with enough symbols (the X is no longer used) has been difficult and the check digit in 13-digit ISBNs do not show up all possible errors generated by either getting a single number wrong or copying down two numbers in the wrong order.

Maths is, like, totally random

Random numbers are very useful in many situations. Random numbers can be generated by physical objects like rolling dice but this is very time consuming and in many situations it is useful to be able to generate random numbers on a computer.

For example many computer games use random number generators to simulate 'real life' so that the opposition players in your football game aren't too predictable or so the baddies don't always come at you at the same time in a shoot out. The lotteries in some countries also use random-number generators to choose the numbers.

Random numbers are also very important in simulations. Modelling the weather, the spread of diseases or the number of people using the queues in a supermarket all involve random events. Mathematical modellers need to generate random numbers to create computer-based simulations so they can try different strategies for dealing with events.

Generating truly random numbers on a computer is impossible but mathematicians have created functions that appear as if they are random. These are known as pseudo-random number generators and prime numbers feature heavily in these functions.

what's the point of...

DISTRIBUTIONS?

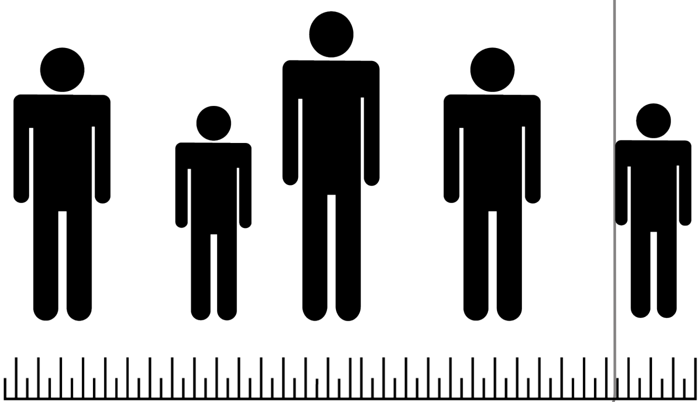
Much of a muchness

What is the average height of students in your class? If your whole class stood in height order, would there be the same difference between each person and the next?

A distribution looks at how a set of data is spread out across different values. Usually the mean is roughly in the middle of the data (you would expect the student with the mean height to be standing near the middle of the line when your class are standing in height order). You can then use mathematics to look at how all the other bits of data are distributed around the mean.

The normal distribution is a very common distribution which describes how most of the data is close to the mean with progressively fewer data points once the values get much bigger or smaller. You can see this in your class where most people are close to the mean height, often just slightly above or below it. There are only a few people that would be considered to be very tall or very short!

The power of distributions is that the same mathematical distributions appears in all sorts of situations: everything from the size of animals and the movement of the stock market to the way heat radiates from objects and the way that people make mistakes! Importantly, if we expect something to be normally distributed but it is distributed in a different way then we can conclude that something suspicious is going on.



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Financial fraud

If you are an investor who wants to invest in an investment fund, you want to make sure that the investment company are being honest when they show you their finances.

However, when an investment company has to release their monthly profit, they are more likely to want to exaggerate a good profit and they would be very keen to cover up a loss. If you plot all of their statements and have a look at the distribution, you can see if it deviates from the shape that you would expect for a good, honest investment company. In fact, in 2009 the Madoff Hedge Fund was part of an alleged £30 billion fraud that was detected using distributions.

HM Revenue & Customs use distributions when checking tax assessments for fraud. They plot all of the assessments for people in the same profession and then investigate any people whose tax deviates from the expected distribution.

Medicine

Much of your body and the way it operates follows well-known distributions. So, if your body starts to deviate from what is expected, doctors know that something might be going wrong.

In medicine, doctors often use what is called a *t* distribution, which is a variation of the normal distribution that is used when there are only a small number of data points. One area this is used in is detecting prostate cancer. If the volume of a prostate gland differs from what is expected, doctors can calculate the chance that this is just a random fluctuation or whether it may be caused by cancer.

The *t* distribution was discovered by William Gosset in 1908. Gosset worked at the Guinness brewery and they didn't allow employees to publish papers in case they gave away any brewing secrets so he published his findings under the pseudonym 'Student'.

what's the point of...

IMAGINARY NUMBERS?

Solving equations

How can you have a number that is imaginary? It sounds like mathematicians are indulging in some wishful thinking!

Actually, when you think about it, negative numbers (and even zero!) are just made-up numbers but they are extremely helpful for describing and solving maths problems. An imaginary number involves the square root of negative one. For hundreds of years mathematicians insisted that you cannot have the square root of a negative number but, in 1545, the Italian mathematician Gerolamo Cardano decided to pretend that there was such a number. (We now call the square root of negative one i.) To his surprise, he found that this new pretend number obeyed the same rules of arithmetic as real numbers and was useful when solving maths problems.

Solving an equation like $x + 5 = 2$ requires you to use the made-up number –7. Solving an equation like $x^2 = -9$ requires you to use the imaginary number 3i. Complicated equations such as $x^3 + 3x^2 - 12x - 18 = 0$ do have real solutions such as $x = 3$ but, to get these answers, during the working-out you use imaginary numbers (in this case, the numbers $2 + 11\mathrm{i}$ and $2 - 11\mathrm{i}$ appear).

Imaginary numbers are like an off-road detour when the normal road is blocked. When you reach a calculation that you can't do with normal real numbers, imaginary numbers can take you off-road around the problem before bringing you back on to the real road on the other side of the blockage.

Getting complex

If a number is a combination of a real number and an imaginary number, such as 2 + 11i, we call it a complex number.

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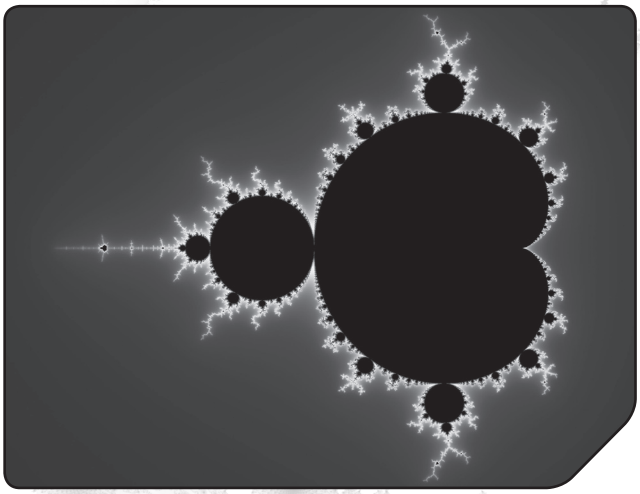
Making pictures

Because a complex number has two parts, one real and one imaginary, it can be represented on a diagram as a point with the real part giving its horizontal position and the imaginary part its vertical position (a bit like coordinates).

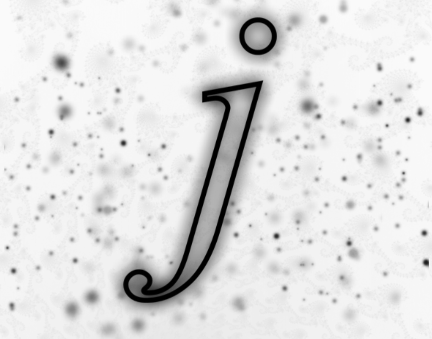
Such a diagram is known as an Argand diagram after the French mathematician Jean-Robert Argand. In 1978 the mathematicians Robert Brooks and Peter Matelski decided to try repeatedly squaring complex numbers and then adding the original number. They then drew an Argand diagram using these complex numbers and coloured in all the ones that stayed small no matter how long they continued the squaring and adding process. What they ended up with is the Mandelbot set, a picture of which is shown on the right.



Electronics

It was noticed by the mathematician Leonhard Euler in 1748 that imaginary numbers were very good at describing things that rotate or oscillate.

Many years later, electrical engineers were trying to find a good way to mathematically represent the alternating currents that power our modern electronic lifestyle. They realised that they could represent the current flowing as the real component of a more complicated imaginary function. While it may sound like this would make things more complicated, doing calculations with the imaginary function was far easier than just using ordinary numbers – particularly when looking at electrical phase change and impedance. At the end of the imaginary calculation, the answer they needed was just the real component.



In electronics, the symbol i was already used, to represent current, so engineers use j to represent the square root of negative one.

what's the point of... INEQUALITIES?

You can stand under my umbrella ...

'Dark rain clouds gather over the horizon, there's one hell of a storm coming.' Actually this has nothing to do with the weather – that's another story altogether – but the old adage of saving pennies for a rainy day has become an increasingly important consideration in recent times.

A variety of factors impact on the investment decisions we make for our future. The population is getting older and the proportion of people in the workforce supporting that population is getting smaller. Where are pensions going to come from? The volatility of stocks and shares can mean that, if there is a stock market crash, the value of an investment can fall to almost nothing. To reduce this risk, investors can choose to invest in less risky financial products that offer fixed (but potentially lower) returns over a period of time.

On the other hand, people may turn to banks for loans to fund immediate purchase decisions such as buying a car, going on holiday, etc. Interestingly, in both these scenarios inequalities are an important factor in the decisions made by the investor or the bank.

If you walk into a bank or apply online for a fixed-rate loan, the first stage is generally assessing your income and expenditure. This acts as a rough and ready measure on your ability to repay the loan. If, after taxes, rent, bills, entertainment, food, etc. your income is greater than or equal to ($\geq$) your projected loan repayments you are likely to be approved for the loan, or at least encouraged to proceed with the application.

Alternatively, when planning for the future, you may wish to receive a fixed return based on how much you invest. Let's say the interest rate at the time is 5% and you want at least £10 000 a year return for when you retire. You can use the calculation $I = Prt$ (where I is income, P is the principal amount invested, r is the rate of interest and t is the number of time periods).

You can say that:

$$I = Prt \geq 10\,000$$
$$P \times 0.05 \times 1 \geq 10\,000$$
$$P \geq 10\,000 \div 0.05$$
$$P \geq 200\,000$$

So, and admittedly this is very simplistic, in order to earn £10 000 a year (before taxes), you would need to have invested £200 000 in a financial product that offers a 5% rate of return. Of course, this does not include factors such as incremental payments, changes in interest rates, etc. but it gives an idea. Start saving those pennies …

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Elasticity – the difference between priceless and pricey

Particularly in the developed world, there are many aspects of our daily lives that we take for granted. Perhaps none more so than the ability to reach for a tap, fill a glass with water and drink it.

The human body can survive almost 18 days without food but only three days without water. Water is essential to life as we know it. No matter how expensive it becomes to obtain, without it we cannot exist. We say that demand for water is inelastic.

On the flipside, as the Christmas holiday season approaches, retailers and manufacturers tempt consumers with powerful advertising campaigns for items such as HD television, the latest games console or designer clothing. We don't need these things, there are plenty of alternatives we could buy. Often, however, it comes down to price. If the demand for a product is affected by the price charged for it, the demand for that product is said to be elastic. Retailers and manufacturers use advertising to try to increase the demand for their goods and make it less sensitive to changes in price. By making it a must-have product they hope that consumers will be willing to pay whatever price is asked.

Retailers and manufacturers may also reduce prices through sales and discounts. With elastic demand, a reduction in price will generate extra demand but will this lead to a fall in revenue? They have to drop the price just enough to attract enough consumers so that revenue will increase.

Price elasticity of demand (E_d) for a given product, is a coefficient that can be calculated using

$$\frac{\text{\% change in quantity demanded}}{\text{\% change in price}}$$

If $-1 < E_d < 1$ demand is inelastic and, if the price decreases, so does revenue (and vice versa).

If $E_d < -1$ or $E_d > 1$ demand is elastic and, if the price decreases, revenue increases.

A variation on the theme of elasticity are aisle promotions and loss-leader products, often found in supermarkets under the banner of 'Buy one, get one free' or '3 for 2'. These incentivised offers are designed to attract customers to buy more of a product but also to encourage them to buy other items at the same time, as well as encourage the behaviour of repeat purchasing from the outlet in the future. The revenue from the products on special offer is likely to be less. Overall, however, the decrease in price of the promotional goods will lead to increased revenue for the supermarket as shoppers start to demonstrate loyalty as well as changes in purchasing behaviour, buying more than they need.


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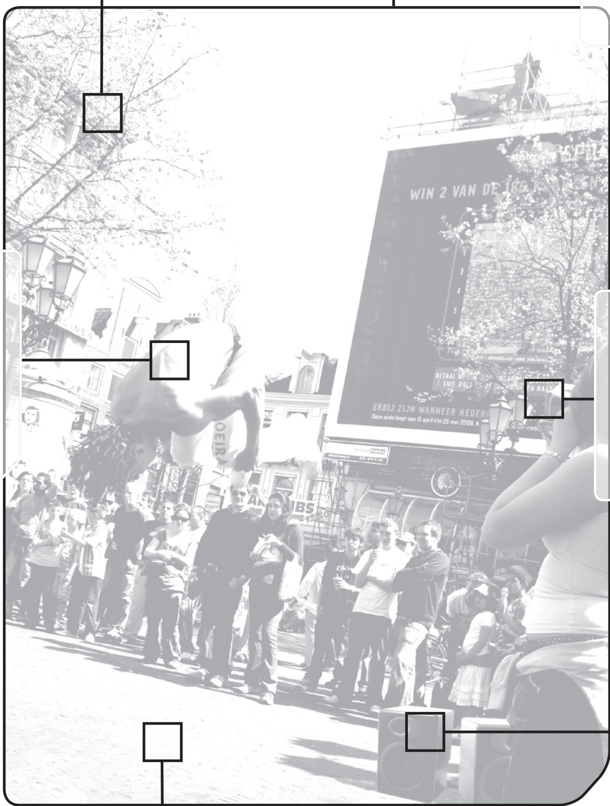
Dear Anne

fractals in the tree

Fractals are geometric shapes with lots of fascinating properties – they also happen to be abundant in nature from snowflake crystals to lightning forks, the structure of the lungs to the branching patterns of trees. Tree trunks have branches, which have branches which have branches and so on. They are all similar but not identical.

crowd safety

The FA Cup Final and fractals represent an unlikely alliance but mathematician Keith Still has found a link. He has showed that people in a crowd do have a sense of direction but, because their line of sight is obscured, they follow other people and compete for spaces as they appear. If the crowd is moving in the right general direction, it is far easier to just go with the flow rather than fight against the tide. Modelling and predicting this behaviour has many applications, including crowd safety.



angular momentum

This dancer has a fixed angular momentum that links his radius to the speed he is spinning. This means that he can tuck his arms and legs in to reduce his radius and make himself spin faster in the air.

camera

A digital photograph involves millions of pixels, each a separate dot of colour. The position of each pixel in the photo is identified using its x and y coordinates

brick tiling

These bricks repeat the same pattern over and over with no gaps. Mathematicians have shown that there are only 17 different types of pattern that will cover the ground like this.

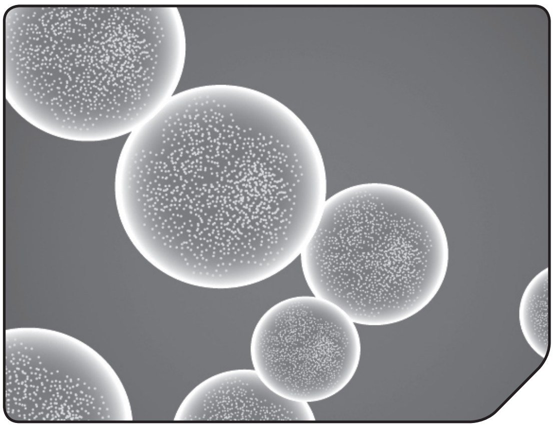
speakers

Digitally recorded music you can hear through speakers or headphones goes through a lot of maths to get to your ears. Fourier transforms take sound waves and translate them into number values (data). A lot of the translated values are outside the human range of hearing so can be dumped, leading to smaller data files. Hence the ability to transport thousands of songs on an mp3 player, while a conventional audio CD will only hold around 20 songs.

what's the point of...

GEOMETRY?

Fighting back against cancer



One of the most deadly yet least understood diseases known to mankind is cancer. Cancer causes cells in the body to grow uncontrollably and the malignant growth may invade other tissues and organs in the body.

Geometry is a branch of mathematics that can help identify cancerous growth and help in the prevention and cure of the disease.

Although cancer cells are three-dimensional, by studying cell structure in two dimensions we can determine whether or not further investigation is required. For example, mammograms are used to screen for abnormalities or to identify the nature of breast lumps.

Cancer cells derive their name from the Greek word for 'crab' because they are often crab- or star-like in appearance. The breast is x-rayed and the cell clusters shown on the resulting mammograms are analysed. If irregular or star-shaped clusters are found, the ratio of

$$\frac{p^2}{a}$$

where p is the perimeter of the cluster and a is the area of the shape, can be determined. The larger the ratio, the greater the concern.

Extending the technique into three dimensions allows comparison of the surface area to volume ratio and offers a different perspective and, possibly, ideas for dealing with cancer.

Investigation

You can investigate the ratio of the perimeter of a shape to its area by taking various measurements of regular shapes (such as squares, pentagons and hexagons) and calculating the ratio

$$\frac{p^2}{a}$$

Extend this to irregular shapes. What do you find?

With regular shapes, you may notice that as the number of sides increases, the ratio decreases. However, with irregular shapes, it is not so easy to generalise and in some cases you will see the ratio increase, meaning further examination of the cell clusters in real life.

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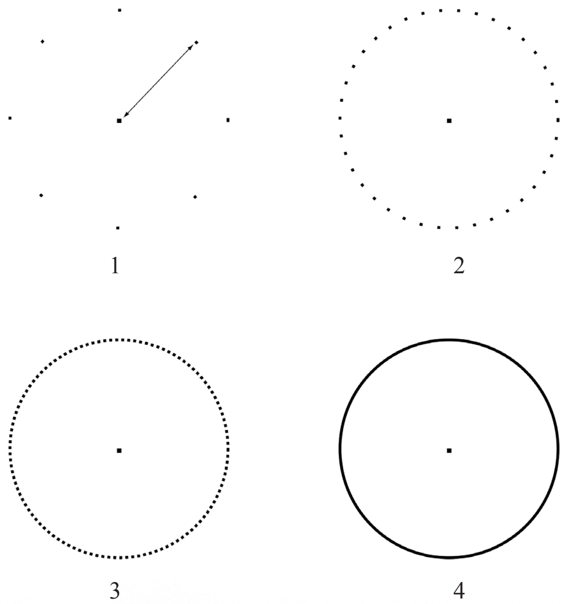
What's that coming over the hill? Is it a locus?

In mathematical terms, a locus is a collection of points that share a common feature or property. For example, a circle is made up of a locus of points that are equidistant from a fixed point (the centre).



This definition can be extended further. For example, the locus of zero values given by a quadratic polynomial (not tackled here!) gives rise to some spectacular shapes called quadratic surfaces with names such as spheres and saddles.

If we delve deeper still we enter the realms of fantasy and can explore impressive geometrical representations such as fractals. Elsewhere in this booklet you can see the Sierpinski triangle and the Mandlebrot set. You can also see fractal geometry in everything from crowd movement at a major sporting event to snowflakes to computer-generated imagery for games and movies (such as is seen in the opening sequence of *Casino Royale*).

There is a simple fractal that you can create that will also give you an insight into the mysterious world of infinity. One mathematician who explored this was Georg Cantor (1845–1918). He produced the Cantor set using the following method.

The Cantor set

1. Start with a line of length 1. This can be defined as the set of points $0 \le x \le 1$.
2. Underneath this draw a copy of the line with the middle third removed

   (i.e. $\frac{1}{3} < x < \frac{2}{3}$ removed.)
3. Underneath this draw a copy of the last line with the middle third of each section removed.
4. Repeat stage 3 (infinitely many times!).

Two questions to think about.

- What is the sum of all the ‘lengths’ of the regions you have removed?
- Can you identify any region or point from the original line ‘length’ that has not been removed?

The sum of all the lengths removed is the infinite series

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \ldots,$$

which totals 1. In other words, you started with a line length of 1 and, if you sum the lengths of all the removed regions, that should also come to 1. You may well be thinking that you must have no points left but, and here’s the brain-frying bit, take the point

$$x = \frac{1}{4}:$$

this point can never be removed using the process above.

In fact there are an infinite set of points that aren’t removed by this process. These are the Cantor set. The Cantor set, by logic, should be empty but, by logic, it should also contain an infinite number of points!

what's the point of...

DIFFERENTIATION?

Go with the flow

Racing cars need to be able to cut through the air at high speeds and buildings need to remain standing even in very high winds. In both situations, engineers need to understand the behaviour of air as it flows around objects.

In fact, air in these situations can be thought of as a fluid and the maths behind how fluids behave is known as fluid mechanics. This covers things as diverse as how blood flows through a heart and how peanut butter moves through pipes in a food factory.

The foundation of fluid mechanics is a collection of differential equations that were derived by Claude-Louis Navier in 1822 and then developed by George Stokes in 1845. This set of differential equations are called the Navier–Stokes equations and they allow us to understand the flow of fluids.

However, even though these equations can be solved to help us understand the flow of anything that can be considered a fluid (and this includes not only gases and liquids but even things like traffic in a city and stars moving in a galaxy) they are not mathematically complete! Mathematicians have not managed to work out if these differential equations will always produce an answer and if any answer they give will always make sense. The Clay Mathematics Institute have decided that this is one of the most important unsolved problems in maths and so they have promised to give a million US dollars to the first person to fix the problem. Could you be the person who gets a million dollars for completing our understanding of the Navier–Stokes–YourName differential equations?

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The need for speed

Conventional methods for detecting speeding include radar guns and roadside cameras. Increasingly, however, average-speed cameras are being introduced to Britain's roads and, with the infallible logic of maths, they are hard to deceive.

Take the example of a driver who has spotted a roadside camera. Obviously if she is speeding and spots a camera, she will slow down using the brakes as she approaches the camera. Once she has passed it, however, she will probably speed up again. If she sees another camera later in the journey, she will apply the brakes again, then speed up again once she's past it.

Now take the scenario where two average-speed cameras are placed at a known distance apart and the driver continues to use the same strategy as before. Using simple distance, velocity and acceleration equations and something called the mean value theorem it is possible to work out whether the driver has broken the speed limit, even if she is obeying the limit as she approaches the cameras.

The position function (or distance) of a travelling car can be mapped as

$$x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

(where a is acceleration, v_0 is initial velocity, x_0 is initial position and t is time). is the first derivative of the equation above with respect to t and this is a measurement of velocity (or speed).

The mean value theorem states that

$$x'(t) = \frac{x(t) - x(t_0)}{t - t_0}$$

(the rate of change in distance over the change in time – in other words the average speed).

In practice, this means that the first camera makes a note of the initial position and time ($x(t_0)$ and t_0 respectively). The second camera marks the finish position and time. By substituting the values into the equation above, it is relatively simple to work out if the driver has been speeding or not.

This method of recording average speed is very difficult to argue against particularly if you want to take it to court. The moral of the story? Drive safely!

Calculus for a healthy heart

When looking for signs of a healthy body, or otherwise, two of the things doctors measure include blood pressure and pulse. The flow of blood around the body is often referred to as haemodynamics.

The flow of blood (and rate of change of flow – differentiation!) through the body is critical to our survival. Often increase of blood pressure and/or restricted blood flow through the arteries (atherosclerosis) due to the deposit of materials such as cholesterol is a major sign that something like a heart attack could strike pretty soon.

The way to measure and monitor such phenomena are based on medical knowledge coupled with mathematical expertise including an understanding of fluid dynamics. Using Newton's laws of mass and momentum, the Navier–Stokes equations, Bernoulli equations, among others, it is possible to model the optimal blood flow through a body.

If the signs are caught early enough you may want to thank maths and medicine, amongst others, for helping to recognise the danger.

what's the point of...

PYTHAGORAS?

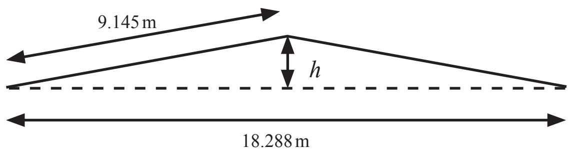
Stay on track with maths

Railway tracks can be made of connecting pieces of rail that are usually 60 feet (18.288 m) long. The pieces are either bolted or welded together at the ends.

When the rails are bolted together a small gap is left between them to allow for expansion in hot weather. The familiar 'clackety-clack' noise is the result of a train moving over the gaps between the rails.

If the gaps weren't left there could be a serious problem with the rails buckling when they expand on a hot day. You can model the effect of a piece of track expanding using Pythagoras' theorem.

If the track expanded by just 0.01% to 18.290 m (i.e. by about 2 mm) the height of the middle of the track would have risen by over 13 cm!



Welded rails are now more common. Welded tracks give a smoother ride, especially at higher speeds, but are more susceptible to buckling as there aren't any gaps. To avoid buckling rails are heated before laying them.

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Want to be square ... use the 3-4-5 rule

You've probably met 3-4-5 triangles when studying Pythagoras' theorem: if a right angled triangle's base is 3 units and its height is 4 units then its hypotenuse is 5 units. This works because the angle between the base and the height is a right angle. Did you know that carpenters and joiners reverse this as an easy way to check if a corner is a right angle?

To check if an angle is a right angle carpenters measure 3 units (inches, metres, tens of centimetres, light years or whatever you want!) from the angle along one side and make a mark, then measure 4 units from the angle along the other side and make a second mark. The distance between the two marks should be exactly 5 units.

This easy check is used to see if edges are at right angles to each other and if walls are perpendicular.

It's not all Greek to me!

Pythagoras lived on the Greek island on Samos between 570 and 495 BC where he brought together one of the first schools of mathematics. He is widely credited with discovering the theorem that bears his name but there are other examples from the ancient world that demonstrate knowledge of the relationship between the length of the sides of right-angled triangles.

In India, where the result is often known as the Bhaskara theorem, there is evidence of knowledge of the relationship that predates Pythagoras. The ancient Indian mathematicians Baudhayana, who lived around 800 BC, and Apastamba, who lived around 600 BC, featured the relationship in their writings. Some people believe that both of these were based on earlier discoveries in Mesopotamia (which covered what is now Iraq, and parts of Iran, Syria and Turkey).

In China the relationship is known as the Gougu theorem. The *Chou Pei Suan Ching* contained a proof of the Gougu theorem and appeared between 500 BC and 200 BC.

what's the point of...

POWERS AND ROOTS?

If maths be the root of music, play on

Did you know that musical scales are based on maths? All notes have unique frequencies, with higher notes having higher frequencies, and the relationships between different notes are mathematical.

The simplest of these relationships is the one between the same note in different octaves: to produce the same note one octave higher you double the frequency. For example the note A above middle C has a frequency of 440 Hz; the note A one octave higher has a frequency of 880 Hz. To increase another octave you would double again, so the next note A, an octave higher is at 1760 Hz. You can show this really easily using a guitar: if you pluck the A string, then press down on the twelfth fret and pluck it again you will have halved the length of the vibrating string which doubles the frequency giving you an A one octave higher!

Music based on one note, even played at different octaves, would be very boring. To generate more notes you need to split the octave into a scale. Traditional Western music uses the Chromatic scale. This has as a basis the frequency 440 Hz for the note A (often written A440) and splits the scale into 12 notes. These are the notes you would see on a piano keyboard or guitar fretboard.

The 12 notes the scale is split into are A, A#, B, C, C#, D, D#, E, F, F#, G, G#. (The # symbol means 'sharp'.) The most common way to generate these notes is to use 'equal temperament' where each step up the scale, or semitone, is defined so that the ratio, r, of one note to the previous note is constant. After 12 semitones you should have moved up a complete octave by multiplying by the ratio 12 times. This means that if you start at A440 you get the equation $440 \times r^{12} = 880$ which solves to give

$$r = \sqrt[12]{2} \approx 1.0595$$

This can then be used to generate the frequencies of all the notes in the scale.

Note	Frequency (Hz)
A	440.00
A#	466.16
B	493.88
C#	523.25
C	554.37
D#	587.33
D	622.25
E	659.26
F	698.46
F#	739.99
G	783.99
G#	830.61
A	880.00

To the human ear the increase in the pitch of the notes sounds constant even though the gap between the frequencies is increasing.

Splitting into 12 is not the only way to split an octave. Arabic music uses up to 24 divisions and Chinese music does not use equal temperament but splits an octave so that the ratio of the frequencies of one note to the next is a whole number. This is often referred to as 'just intonation'.

The relationship between maths and music is very rich and this is just one example. There are many other mathematical features that occur in music such as Fibonacci numbers and the golden ratio.

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Slide rule

You've probably noticed skid-marks left on roads where cars have had to brake suddenly. Accident investigators can use maths to tell the speed of a car from the length of the skid mark.

The length of the skid mark is proportional to the speed of the car squared. So investigators use the formula: $d = kv^2$ where d is the length of the skid-mark, v is the speed and k is a constant that depends on such things as the road surface and the conditions.

Rearranging this gives:

$$v = \sqrt{\frac{d}{k}}$$

which lets them estimate the speed directly.

There are various reasons why this might be an underestimate of the speed though. Only part of the skid marks may be visible, a collision may have affected them or the brakes may have slowed down the vehicle before the car started skidding. For these reasons the speed calculated is often referred to as the minimum speed, leaving the accident investigators with an inequality, and often more maths to do!

Maths keeps the world in motion

Johannes Kepler lived in central Europe, in what is now part of Germany, between 1571 and 1630. He was an astronomer and mathematician who studied the motion of the planets and was one of the first people to write in defence of Copernicus' model of a sun-centred (or heliocentric) universe; before this most observers believed the Earth was at the centre of the universe. This defence took him eight years to perfect!

Kepler studied data collected from observations by the Danish astronomer Tycho Brahe and suggested three laws for planetary motion. The third of these laws relates the time it takes for a planet to orbit the Sun (its period) with its distance away from the Sun.

Kepler's third law states that the square of the period, P, is proportional to the cube of the distance, d:

$$P^2 \quad d^3 \text{ or } P = kd^{1.5}$$

where k is constant.

Kepler even recorded the day he made the discovery: 15th May 1618, although he did also state that this discovery was a result of seventeen years of hard work – there's a lesson for us all there!

what's the point of... EXPONENTIALS?

Saving lives with maths

Many diseases are caused by viruses. In order to help stop the spread of viruses, from those causing the common cold to much more serious ones like HIV, it is necessary to be to able predict how fast they will spread to.

It is the job of a mathematical biologist to create a mathematical model of how many people can expect to be infected. The spread of many viruses can be modelled by exponentials. This is because the number of people who are infected in any given time period is usually proportional to the number who are already infected.

An accurate model is essential in deciding how to try to contain viruses. Modelling the spread of a virus as exponential growth is especially accurate in the first stages of an outbreak. It is then possible to adapt this to give more accurate predictions as the virus spreads further. These models can be used to test different strategies for dealing with the disease using computer-based simulations. These strategies can include the use of vaccinations, other drugs or even travel restrictions.

Using an accurate model can even help in eradicating viruses completely. The model can be used to predict the proportion of a population that needs to be vaccinated so that the virus does not spread at all – this is known as herd immunity. Smallpox has been eradicated worldwide by vaccination; polio is likely to be eradicated soon by this method.

A model is only as good as its assumptions (and the mathematics!).

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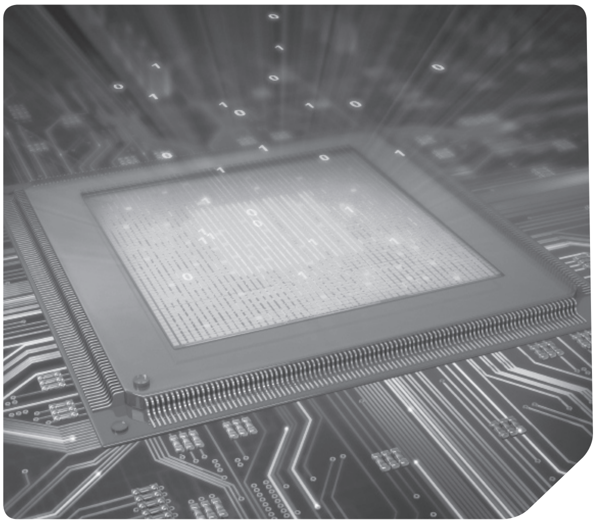
The pace of change

In 1965 Gordon E Moore, one of the founders of Intel, suggested that the number of transistors that can be fitted onto a silicon chip doubles every two years. The number of transistors affects many aspects of computing such as power, speed and capacity and consequently all of these qualities have grown exponentially.



Although the law was first suggested over 40 years ago it still remains accurate today. In 1972 it was feasible to fit 2500 transistors on a silicon chip, by 1974 the figure had doubled to 5000. By 2008 the figure was nearly 2 billion!

The law is now seen as a standard that all producers of computer hardware should try to achieve.

There is much debate about how long the exponential growth predicted by Moore's law can continue. As transistors get smaller and smaller the production techniques required to make the components get more difficult. Some current components are only a few atoms thick and it is possible that as these are reduced further then a limit will be reached. The world of computing relies on mathematicians and scientists to devise alternative technologies to produce hardware that produces the constant improvements that the modern world has come to rely on.

Using maths for dating

It's not just exponential growth that's useful. Exponential decay is too and can be used to estimate the age of ancient objects.

Carbon-14 is a radioactive isotope of carbon (the more common isotope being carbon-12). The ratio of carbon-14 to carbon-12 in the atmosphere has stayed almost constant for over 50,000 years at about 1 part in a trillion. This is the ratio found in any living plant or animal. Once a plant or animal dies it stops taking in new carbon and the carbon-14 decays radioactively, so the proportion of carbon-14 reduces exponentially. The half-life of carbon-14 is 5730 years. In this time the amount of carbon-14 reduces by a factor of a half. Using this fact the proportion of the carbon in an organic object that is carbon-14 can be used to determine the age of that object.

One famous example of an object that has been carbon-dated is the Turin Shroud. This is a piece of linen cloth found in a chapel in Turin Cathedral in Italy. The cloth appears to show the image of a man who has been crucified. Many people believe that it is the cloth placed around Jesus after his crucifixion. Radio-carbon dating in 1988 showed the cloth to be about 700 years old. There is still much controversy over its true age.

GRAPHS?

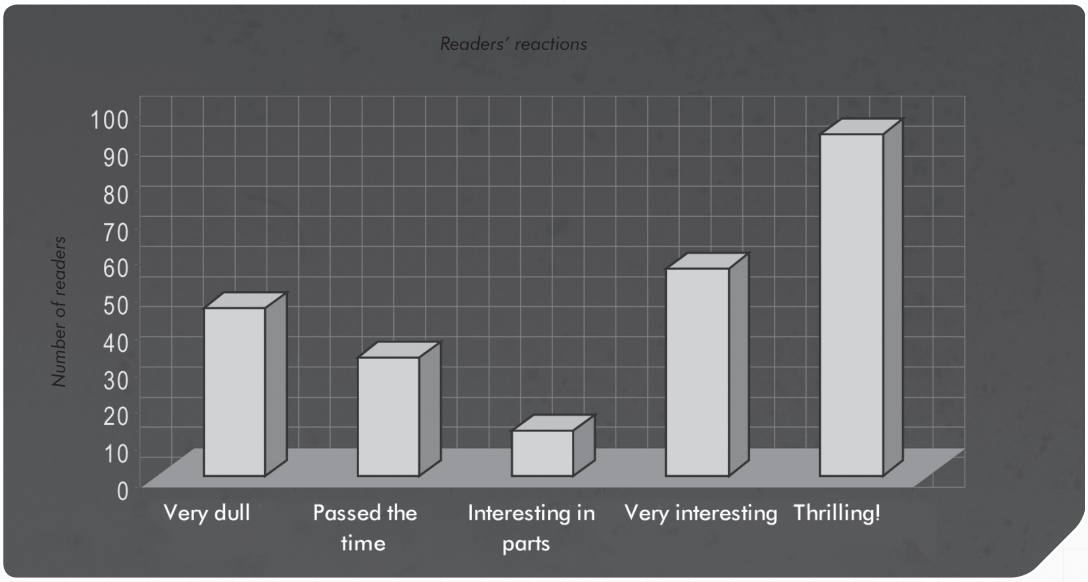
See it my way

We like pictures, they help us understand things. About half your brain is working to let you see things, so it's no surprise that humans have a highly-developed visual sense.

That's why graphs are so vital. Rather than having to get your head round a long list of numbers you can use them to draw a graph. Graphs let you see how the numbers are related: are they going up, down, up then down? Whatever the trend, a graph can let you see it almost instantly. Graphs are encountered in many jobs so it is likely that you will have to get to grips with graphs – they are everywhere.

Graphs turn up on TV all the time. On the news they can show the rate of inflation for the last 10 years or the increase in arctic ice melt. On a game show a graph can show the number of audience votes for each possible answer. On a consumer affairs programme you might see a large foam meat pie divided up to show the proportions of fat, sugar, protein and salt in a typical pie, each pie slice being in proportion to the amount of pie stuff it represents.

You need to be able to understand graphs to be able to understand the world around you, from newspapers to scientific papers, from magazines to TV shows, graphs are the picture-perfect way to present data.



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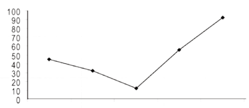
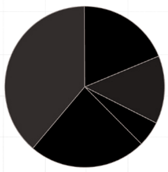
All kinds of everything

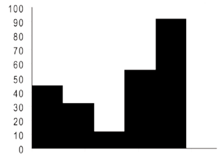
There are many different types of graphs, often called charts, but each is just a different way to turn numbers into patterns to help us to understand them more easily. Typical kinds of graphs include line graphs, pie charts and histograms.



Line graphs have vertical and horizontal axes and points are plotted and joined with lines. The finance markets use these all the time.



In pie charts you turn the numbers into slices (sectors) of a circle, the size of each slice being dependent on the relative size of each of the numbers. Businesses love these sorts of charts as it's easy to see how much of each of their products they are selling.



Histograms are another popular type of graph. Here you group together similar values into 'bins' so, for example, a bin might have size 5 so that any values between 0 and 5 should be added into the first bin. In the final graph the number of values in each bin is plotted as a bar, the area of which is related to the amount of stuff in each of the bins. Histograms are a good way to present a wide range of data and the frequency with which these groups of values turn up, without the detail of a line graph, so histograms are very popular in business too.

Computers today will allow you to take your spreadsheet data and turn it into all sorts of different types of graphs, so you can choose the one that is best for your presentation, but of course you need to understand the basics, or you could end up showing rubbish.

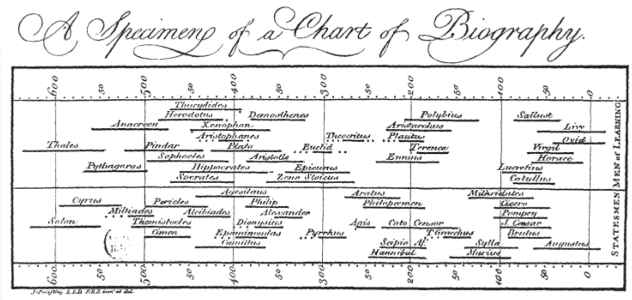
Graphical history

Graphs are mathematical tools and someone had to invent them. The histogram was first introduced by the statistician Karl Pearson in the 1890s. As well as inventing and using graphs, he also found time to set up the first ever university statistics department.

The word histogram comes from the Greek words 'histos' meaning 'anything set upright' (here this refers to the upright bars on the graph) and 'gramma' meaning 'drawing, recording or writing'.

The pie chart was created by Scottish mathematician and engineer William Playfair around 1800. He was a serial graph inventor having also created the line graph, the bar graph and the little-known circle graph. Graph inventors were quite the celebrities in their day.

Historian, scientist and philosopher Joseph Priestley created timelines in the 1760s. These used bars to compare the lifespans of various people and civilisations. They proved to be a commercial success and were reprinted many times. They were probably the inspiration for Playfair's later graph inventions. These early types of graphs made it easy to see the points that were being made and became a national sensation. Priestley was even awarded a Doctor of Law degree for his wonderful lectures and graphs.



Priestley Chart

what's the point of...

VENN DIAGRAMS?

Logic, probability and programming

What would you do if, at the end of the school assembly, all of the members of the football team had to wait behind for a message and all the people in the maths club had to meet outside the hall in the foyer and you were in both? Would you stand in the doorway, half in and half out of the hall? If you did, you would be part of a human Venn diagram!

Venn diagrams were first put forward by John Venn in 1881 as what he called a 'diagrammatic representation of propositions and reasonings'. In short, he was looking for a good way to draw logic statements instead of writing them all out as complicated sentences.

If you had the three statements:

- I am a member of the football team
- I am a member of the maths club
- I am not a member of the knitting squad

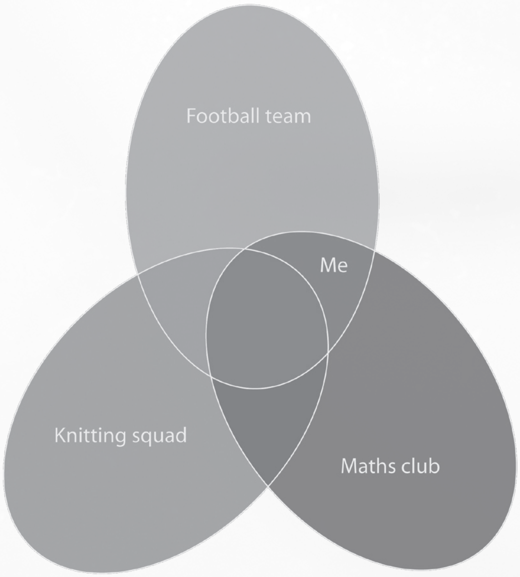
you could represent all of them at once by drawing a circle to represent each of the groups, overlapping them and showing where you sit.

There are eight different regions in this diagram (including the region that is outside of all the circles) so that every combination of which of the groups you are or are not in can be shown. The power of the Venn diagram is that it can take a complicated list of rules about how different objects belong in various groups and convert it into one picture.

In the 1800s a mathematician called Augustus de Morgan stated what are now known as de Morgan's laws and these underpin all of the logical statements that are used in modern computer programs. If you study statistics, you will meet notation to represent a logical statement, such as

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}$$

However, this is much easier to follow when it is represented as a Venn diagram. This is exactly what modern computers do when they are processing information according to the rules of logic.



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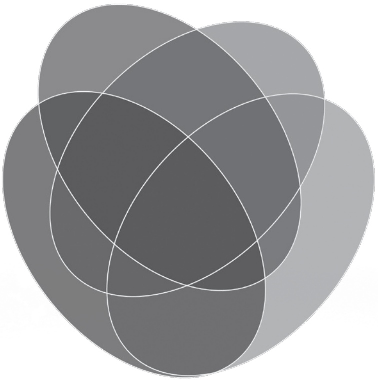
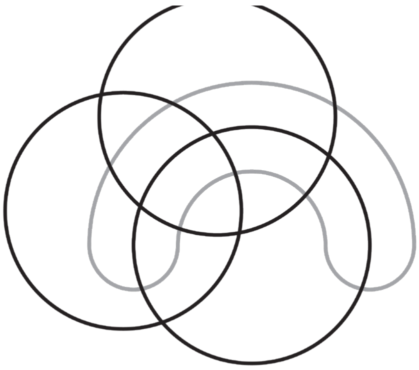
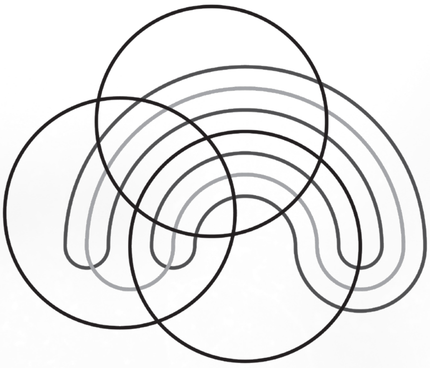
You can draw Venn diagrams for different numbers of sets but, as the number of sets increases, the diagrams become increasingly complicated.



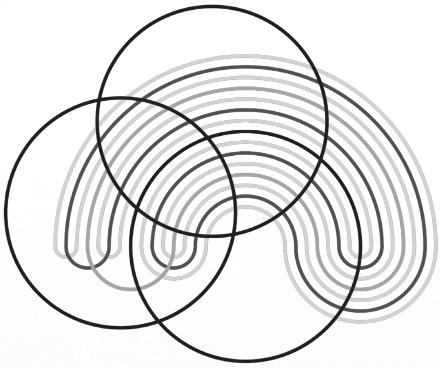
Venn's four-set diagram using ellipses



Venn's construction for 4 sets



Venn's construction for 5 sets



Venn's construction for 6 sets

Genetics

In genetics, it is said that a gene is being expressed if it 'switches on' and starts producing the protein that the gene codes for.

To help understand what the various genes are responsible for it is important to be able to look at which genes are expressed and which aren't in different situations. Geneticists use something called a microarray to analyse a genetic sample and produce a long list of which genes are being expressed. It would be a complicated task to compare the genes that are being expressed in one situation with those being expressed in other situations, but Venn diagrams can come to the rescue! Geneticists can use a computerised Venn diagram program to show them all of the overlapping gene lists.

what's the point of...

NUMERICAL METHODS?

Computer games

Maths is all about investigating the patterns and relationships in the world around us and mathematicians love looking for deeper meaning and logic.

While the mathematicians will play around with the algebra of an equation, impatient engineers just want to know what the answer is! When you need the answer to an equation quickly and you don't need it to be absolutely accurate, numerical methods can help you out.

There are algebraic ways to solve the equation $x^2 - 5x + 3 = 0$ but, if you just try a few numbers using your calculator, you can work out fairly quickly that the first answer is between 0.69 and 0.7 (and that it's closer to 0.7). If you only need the answer to one or two decimal places, this is a quick way to get a fairly accurate value without using algebra.

Computer games use a huge number of mathematical equations to describe the objects in the game and how they interact. These equations have to be solved and the results rendered into a 'frame' to be sent to the screen at least thirty times a second. This means that your computer needs to solve all the equations involved in a 30th of a second! But, because the answers only need to be accurate to the nearest pixel, numerical methods are used to mathematically generate each frame of the game within that fraction of a second.

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Engineering

In engineering, mathematical models are used to simulate everything from new Formula One cars to proposed sky scrapers. The more factors included in the equations (such as air resistance, distributions of mass and the elastic properties of materials), the more accurate the models.

However, more factors make it much harder to calculate any answers from the model. Using numerical methods means that extremely detailed systems can be modelled and meaningful outputs calculated. The beauty of numerical methods is that you can refine your answer to increase its accuracy to whatever precision you require.

At school you will come across different numerical methods, from just guessing numbers to quite complicated algorithms. Some of these work better than others in different situations and mathematicians are always trying to find better and more efficient numerical methods. There is even an International Center for Numerical Methods in Engineering that organises research into numerical methods to help support advances in engineering.

One of the oldest numerical methods is called the Newton–Raphson method. The method was originally suggested by Sir Issac Newton in 1669 and then, in 1690, Joseph Raphson expanded it into the iterative form that is still used today.

Before the advent of modern computers, numerical methods were used with the aid of huge books filled with tables of values for different functions. When a mathematician was calculating a numerical solution, they could look up pre-calculated values that had been listed with up to sixteen decimal places.


G-WDEB
Bassetts

buoyancy

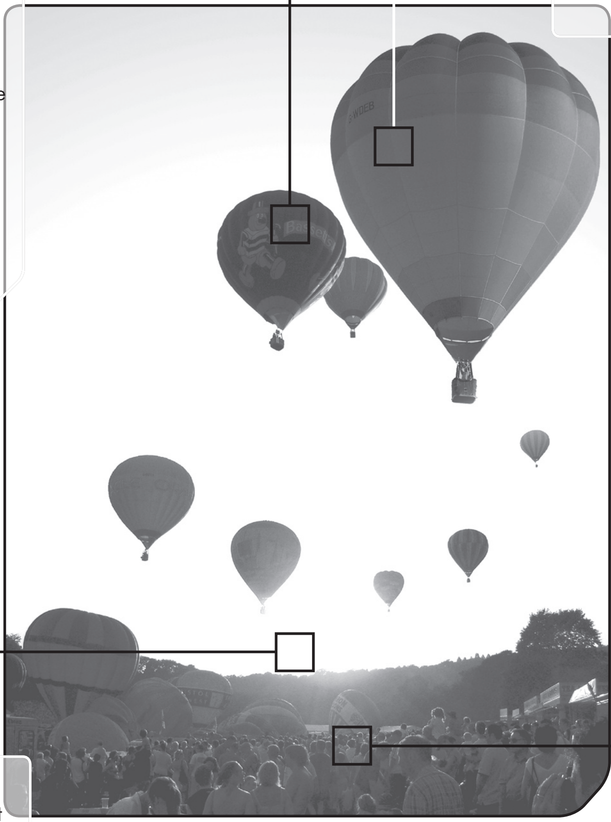
Buoyancy is a physical phenomenon that was discovered by the Greek mathematician and inventor Archimedes in the third century BC.

Since air pressure decreases with height, air presses harder against the bottom of a balloon than against its top. This difference in pressure creates an upward push. As long as this upward force is greater than the balloon's weight, the balloon will rise. But as it rises, the density and pressure of the air around it decreases, so the buoyant force decreases too. When the balloon reaches a height at which the buoyant force equals the balloon's weight, it stops rising.

Hot-air balloons use heated air to provide buoyancy, because warm air is less dense than cool air.

material science

These balloons aren't made of rubber; they're constructed from Ripstop Nylon. Material scientists have developed this synthetic fabric and used maths to ensure that it is both light enough and strong enough whilst still preventing air from passing through it.



sun light

Scientists have used maths to measure that the speed of light is 300 million metres per second and that the Sun is 150 billion metres from the Earth. This means we can calculate that this light left the Sun over 8 minutes ago.

sunscreen

The Sun emits UV radiation. People who go unprotected in the sun can suffer sunburn and increase their risk of developing skin cancer. Sunscreen products protect against UVB radiation, which causes sunburn, and increasingly also protect against the harmful effects of UVA radiation. The sun protection factor (SPF) of a sunscreen gives a rough guide as to how much longer you can stay out in the sun before burning. A sunscreen with SPF 15 allows you to remain in the sun 15 times longer than if you were unprotected.

what's the point of...

INDEX BALLOONS

maths careers
endless opportunities

more maths grads
multiplying opportunities

what's the point of...

INTEGRATION?

You're a Greek philosopher in the year 225 BC. What's the area of a circle with given radius?

You're a wine merchant in Austria in the year 1615. Which shape of barrels will hold the most wine?

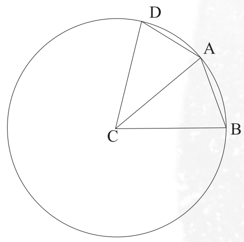
You're designing a new type of airbag to prevent head injury in car crashes in 1955. Does it work?

You're a particle physicist in 1989. How much force do you need to separate two electrons?

You need to create a better version of JPEG compression for image files. What maths will be useful?

Integration helps us to answer each of these questions. Integration is closely associated with its opposite process, differentiation. Together they are known as calculus. Related ideas have been studied for at least two thousand years. The idea of integration is based on calculating an area or volume by adding up lots of small areas or volumes that are easier to compute.

Suppose you have a circle with radius r and you've forgotten that the formula for its area is $A = \pi r^2$. You could work out the area roughly by filling the circle with triangles and calculating the area of each triangle. This is what Archimedes did over two thousand years ago to work out a better estimate for the value of π.



Give me a place to stand and I will move the earth

One of the greatest mathematicians of all time, Archimedes was born in Sicily in the Mediterranean in 287 BC and was killed in the Roman invasion in 212 BC. In between he figured out a huge amount about mathematics and physics, and designed a water pump that is still in use in Egypt today.

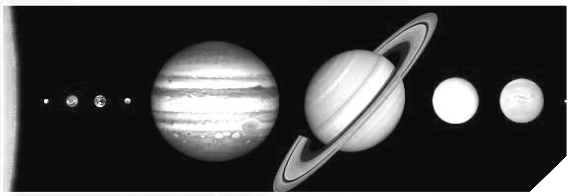
He once said to his friend King Hiero, "Give me a place to stand and I will move the earth." The king challenged him on this. Archimedes then chose a ship which needed many men to move it out of the dock, set up a pulley, and was able to move it himself without much effort.

Archimedes also showed that the exact value of π lies between the values $3^{10}/_{71}$ and $3^{1}/_{7}$ by drawing two regular polygons with 96 sides, one inside a circle with its corners on the circle (inscribed) and one outside the circle with its sides just touching the circle (circumscribed). Modern integration was born out of ideas like this.

Eighteen hundred years later…

Johannes Kepler lived in central Europe. He worked on data gathered by the Danish astronomer Tycho Brahe and figured out that the planets moved in elliptical – not circular – orbits around the sun. This is why sometimes Pluto is closer to the sun than Neptune – its orbit is more squashed.

He noticed that planets travel faster at some points on the orbit. The line joining a planet to the sun sweeps out the same area in a given interval of time, no matter where the planet is. This means that the planet must move faster when it is closer to the Sun.



At his second wedding, Kepler got distracted trying to figure out a better way to work out the volume of the wine barrels there. He wrote a book on the subject in 1615.

In both these problems Kepler used the idea of splitting up an area or volume into smaller parts in order to compute it. This is the key idea of integration.

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Preventing injury in crashes

You're travelling in a car along a city street at 30 mph. What happens if you have to brake suddenly?

Usually it takes 1.5 to 2 seconds to stop a car when braking normally. However in a violent impact, such as a car crash, it can take as little as 0.1 seconds to stop a car. This can cause serious head injuries.

Since the 1950s, many cars have come equiped with airbags in the dashboard. These help prevent head injuries by slowing down the deceleration of the people in the car.

In tests of airbags, a calculation is made called the Head Injury Criterion, or HIC for short. If the test gives a HIC value above 1000 then the crash would have been life-threatening. Modern cars may have HIC values of 100 to 200. The HIC is calculated by looking at every possible time interval between start time, r, and stop time, s, during the braking period and finding the average deceleration for each of those time intervals. To find the HIC we take this average deceleration raised to the power 2.5 (based on car crash data) and multiply it by the length $(s - r)$ of the time interval. The HIC is the maximum over all possible time intervals $[r; s]$.

How do you find the average deceleration? It is the integral of the deceleration, divided by the length of the time interval.

The deceleration at time t can always be found, either by integrating or by approximating the area under the curve at that point.

Now imagine the maths that Formula One engineers use to make sure their cars stay on the road even when travelling at 200 mph!

Keep it down!

There are many more applications of integration and of calculus. The JPEG 2000 image compression standard is based on wavelet theory which uses a lot of integration. Image compression ensures your photo files take less memory per image.

Calculus is needed in physics to calculate the effects of forces on tiny particles or in massive galaxies. Economists use integration techniques to model stock prices.

Integration equips you with the essential skills necessary for either a technical or scientific profession!

Websites to check out:

www.mathscareers.org.uk
plus.maths.org

Interview with maths student:
"If I've got a maths degree, I can be pretty much anything!"
http://plus.maths.org/issue28/interview/index.html

The MacTutor History of Mathematics Archive at the University of St Andrews:
turnbull.mcs.st-and.ac.uk/history/

what's the point of...

LOGARITHMS?

Disaster prevention: understanding earthquakes

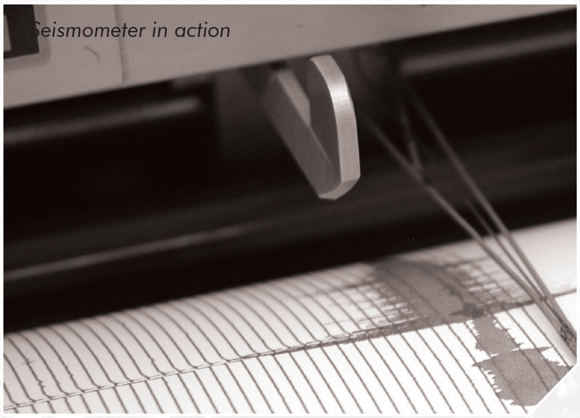
On the 8th October 2005, a major earthquake struck a mountainous region of South Asia. The shock waves radiated out from the epicentre of the earthquake, about fifty miles north-east of Islamabad, the capital of Pakistan.

It wiped out many villages and left over three million people homeless. Over seventy thousand people died in Pakistan and in the Indian-administered state of Jammu and Kashmir.

On the 23rd September 2002, a minor earthquake hit the United Kingdom. The epicentre was in Dudley in the West Midlands, north-west of Birmingham. A few homes were damaged but no-one was injured.

How much stronger was the first earthquake than the second?

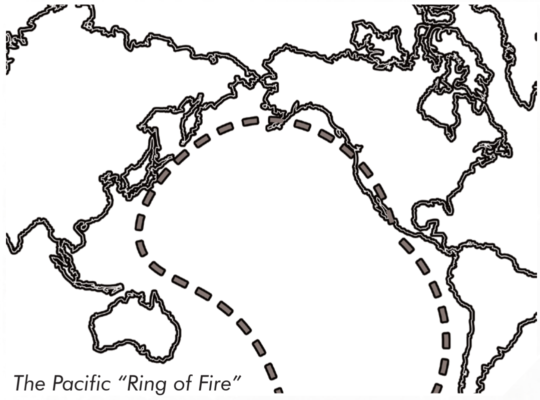
You can measure the strength of an earthquake by using a seismometer. The seismometer measures how much the earth shakes and records it as a graph. Stronger earthquakes have graphs which go up and down more: you can say that the maximum difference in height, which is called the amplitude of the graph, is bigger. This amplitude tells you how strong the earthquake is.



Seismometer in action

Where do earthquakes happen?

Nine out of ten earthquakes happen along the Pacific Ring of Fire, which circles the Pacific Ocean. Japan, California, Chile and the Philippines all lie along this ring. Seventy years ago two earthquake scientists, Charles Richter and Beno Gutenberg, were working in California. They wanted a way to tell how many of the earthquakes in California would be big ones causing serious damage. They decided to give each earthquake a magnitude number. An earthquake with a higher number would be more serious than one with a lower number. The earthquakes mentioned earlier were measured at 7.5 (South Asia) and 4.8 (UK).



The Pacific "Ring of Fire"

How do you calculate the magnitude of an earthquake?

These numbers are calculated by taking the amplitude of the largest wave, taking its logarithm to base 10, and then adding a factor which depends on the distance between you and where the earthquake is. Because the scale is created by taking logarithms to base 10, an earthquake with magnitude number 7 will be ten times stronger than a magnitude 6 earthquake.

How much stronger was the Asian earthquake?

We take the difference between their magnitude numbers and get 7.5 – 4.8 = 2.7. Therefore 2.7 is the logarithm to base 10 of the number we want. If we calculate 10 to the power 2.7 on a calculator we get 501.19. Try it out for yourself. This means that the Asian earthquake was five hundred times stronger than the one in the West Midlands.

Why do people use logarithms here?

It's much easier to talk about earthquakes with magnitude 6.5 or 9.0 than to talk about earthquakes with 5 000 000 or 32 000 000 000 tons of energy.

US Geological Survey's About Earthquakes page, animations, facts and photos: www.earthquake.usgs.gov/learning/

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Apple juice, coffee, milk and soap

Another scale which uses logarithms is the pH scale which measures how acidic a liquid solution is. An acid such as vinegar has a pH value of around 3.

The opposite of an acid is an alkali. Alkalis include soap and bleach. Chemically, an alkali cancels out an acid. Since many stains on clothes are acidic – tea, coffee, apple juice, milk – washing powders and bleaches are usually alkaline. Household bleach has a pH value of around 12.5.

Somewhere in between 3 and 12 on the pH scale we find solutions with a pH of 7. The pH of pure water is 7. Anything with a pH of less than 7 is called an acid; anything with a pH of more than 7 is called an alkali.

Just as for measuring earthquakes, this scale is logarithmic. This means that an acid such as lemon juice with a pH of around 2.5 is ten times more acidic than an acid such as orange juice with pH 3.5. Even your skin is slightly acidic. The soap in your bathroom probably has a pH value of between 9 and 10 so it'll help remove the sticky orange juice but won't react much with your skin. The bleach would be about a thousand times stronger, which is why you don't put it directly on your hands!

Once again, using logarithms helps us use a scale of numbers which is faster to write down.

Experiment

Get a can of cola and some dirty 1p and 2p coins. Leave the coins in a glass of cola overnight. Next morning take your coins out of the glass. The acid in the cola will make your coins look new and shiny! Why? Cola contains phosphoric acid – it's as acidic as lemon juice!

Interesting times

How much does your favourite snack cost? It probably costs a bit more than it did a few years ago. This is due to inflation – in a healthy economy prices creep up slowly. To make up for this, employers usually give their employees a cost-of-living increase in their wages each year.

What about people who save money? Banks will pay interest on your savings so that they also increase in value. They might pay it monthly, or every three months, or once a year. Which is best?

Suppose that you have £5000 in the account and the bank pays 5% annual interest, and computes it every six months. After six months you would have £5000 $\times \sqrt{1.05}$ = £5123.48. After a year you would have £5123.48 $\times \sqrt{1.05}$ = £5250.

What if banks calculated interest differently, finding the interest paid every six months by halving the annual interest rate? How much would you have after three years?

£5000.00 × (1 + 0.05 × ½) = £5125.00 after six months.
£5125.00 × (1 + 0.05 × ½) = £5253.13 after one year.
£5253.13 × (1 + 0.05 × ½) = £5384.46 after 18 months.
£5384.46 × (1 + 0.05 × ½) = £5519.06 after two years.
£5519.06 × (1 + 0.05 × ½) = £5657.04 after 30 months.
£5657.04 × (1 + 0.05 × ½) = £5798.47 after three years.

0.05 corresponds with the 5% rate. We also multiply by ½ because six months is half of a year. The interest is rounded to the nearest penny. The final amount is £5798.47. What would happen if the bank computed your interest every month, or every day?

Final amount after three years if interest is paid on £5000 or on £10 000.

INTEREST PAID EVERY:	INITIAL AMOUNT £5000	£10 000
One year	£5788.13	£11576.25
Six months	£5798.49	£11596.93
Three months	£5803.84	£11607.55
Each month	£5807.54	£11614.72
Twice a month	£5808.66	£11616.53
Every day	£5809.11	£11618.22
Every hour	£5809.17	£11618.34
Every minute	£5809.17	£11618.34
Every second	£5809.17	£11618.34

If interest is paid more frequently, you get more. However, after a point, the extra amount gets so small as to not make a difference. Computing the interest over increasingly smaller time intervals does not result in any extra money. The maximum value you can get is the original amount multiplied by 1.161 833 7. If you take the logarithm of this to the base e (where e = 2.718…) you get 0.15, which is 3 x 0.05 (number of years multiplied by the interest rate). This is true for any period and any interest rate. Logarithms are used a lot in investment banking for making financial calculations like this.

The number e, which equals 2.718 281 8…, is special in mathematics. It was first discovered in 1683 by Jacob Bernoulli, a Swiss mathematician who wanted to understand the compound interest problem. But it is also special because the function $y = e^x$ differentiates to itself, and for many other reasons.

Websites to check out:

www.mathscareers.org.uk
plus.maths.org
Interview with a financial engineer:
www.plus.maths.org/issue46/interview/index.html
History of the number e and of logarithms:
www-history.mcs.st-andrews.ac.uk/HistTopics/e.html

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what's the point of... PROBABILITY?

Oh no...penalties...again!!!

In the summer of 2008, football fans could follow Euro 2008 without the stress of seeing any of the home nations knocked out on penalties (because they never managed to qualify in the first place).

Take England. Out of the last eight major tournaments that they have qualified for they have gone out on penalties five times (being knocked out by other means the other three times). This raises an interesting question – as the opposition manager about to play England, should you play for penalties?

In total, England have been involved in seven penalty shoot-outs in competition and have lost six of them – their only success coming against Spain in Euro '96. So is this 14% success rate statistically significant? How can England improve the odds of success in penalty competitions? Penalties are supposed to be a hit and miss affair – but with a bit of practice and some mathematical analysis, England may well overcome their penalty-taking curse.

Let's set up a simple scenario when taking a penalty.

- A striker can shoot either to his/her left or right, and similarly a goalkeeper can dive to his/her left or right.
- If the goalie dives to his/her left and the striker shoots to his/her left OR if the goalie dives right and the striker shoots right then a goal is scored (assuming the striker is accurate) because the goalie will be diving away from the ball.
- If the goalie dives to his/her left and the striker shoots to his/her right (or vice versa) then the goalie and the ball are reasonably close together and there is a 50% chance the goalie will save the ball.
- Let's assume that the striker is accurate when shooting left 70% of the time and 90% when shooting right.

Using mathematics we can estimate the best strategy for the striker to employ – it involves shooting to his/her left 56% of the time and to the right 44% of the time, irrespective of the goalkeeper's strategy. Overall this corresponds to scoring around 60% of the time. But why should the striker shoot more to his/her left side even though this is less accurate (70%) than when shooting to the right (90%)?

Using the same mathematics we can also estimate the best strategy for the goalkeeper – it suggests diving to his/her left 69% and to the right 31% of the time. So if the striker shoots to the more accurate right side, the goalkeeper will dive more often to his/her left and increase the chances of saving the shot. However if the striker shoots to the less accurate left side, the goalie will only dive in this direction (to his/her right) around 30% of the time – so the lower shot accuracy is compensated for by the fact the shot is less likely to be saved because of the goalkeeper's strategy.

(For a more in-depth perspective on the maths, please see the article by John Haigh on *Plus* magazine website: http://plus.maths.org/issue21/features/haigh/index.html)

Of course, penalties are blasted into the back of the net or accurately placed. They may be in the top left corner, straight down the middle or in the bottom right corner. The goalkeeper may elect not to dive at all or may find that reaching a penalty to the top left is more difficult than reaching a penalty aimed to the bottom left. But at this stage you simply construct a more realistic model involving more than just shooting left and right.

So practice is the better alternative, but the maths and statistics can help analyse performances. In fact, think of all the stats that underline a good performance – not just penalty taking – the distance covered by Steven Gerrard in a match, the number of tackles by Cesc Fabregas, the pass accuracy of Lionel Messi or the power of a shot by Cristiano Ronaldo – it all counts …

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The long arm of the law - probably

In 1999, Sally Clark was tried, convicted and sentenced to life imprisonment for the double murder of her two sons who were aged just 11 weeks and 8 weeks at the time of their deaths.

The tragedy shocked the nation, as the expert testimony of Professor Roy Meadow indicated that the chances of the double deaths happening in the same family from natural causes – Sudden Infant Death Syndrome (SIDS) commonly known as cot death – were 1 in 73 million. In other words, so unlikely that Sally Clark must be guilty of the murder of her sons.

However doubts surfaced about the testimony of the expert witness on the grounds of poor mathematical reasoning. The Clarks had always protested their innocence and there was much debate about the testimony; the Royal Statistical Society had issued a press release pointing out the mistake and indeed the conviction was quashed in 2003.

So what happened? If two events are considered to be unconnected they are said to be independent of each other. Professor Meadow made the (invalid) assumption that the two cot deaths were independent. For a non-smoking, affluent family the chance of a cot death occurring is around 1 in 8500. So to calculate the probability of two deaths occurring in one family he simply multiplied the probabilities together giving a result of 1 in 73 million. He then presented this as the probability that Sally Clark was innocent. This is a case of the Prosecutor's Fallacy. Are you guilty given the evidence or given the evidence are you guilty?

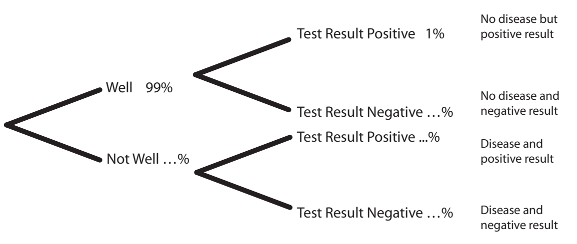
However, research suggests that in a family where one episode of cot death has occurred, the chance of it happening to another sibling is increased by between 10 to 22 times – this means that two cot deaths are certainly not independent. Also consider this, in normal circumstances the probability of either double SIDS or double murder in a single family is very small but, given that a double death has actually occurred, the chances of it being double SIDS or double murder are more likely.

Is there maths in that too? Probably

Medicines that come to the market have done so on the basis of rigorous testing and statisticians are vital to that role.

Pre-clinical trials produce masses of data that must be carefully analysed to determine safety. Clinical trials involving people can take a number of years and include the design of safe trials, the right dosage of medicine and other factors.

Suppose we undertake a screening programme to identify a disease and hence administer a cure. The aims are quite reasonable. Now suppose 1% of the group suffer from the disease and the rest are well but also that the there is a 2% chance that the test produces a false result. Using this information can you complete the following probability tree diagram?



By moving along the branches we can calculate the various probable outcomes and fill in the probabilities associated with each outcome. The two 'dodgy' outcomes are small enough to be considered acceptable. The probability of being well but having a positive test result is known as a False Positive, and the probability of having the disease but having a negative test result is known as a False Negative.

However, in real life the medication we need to administer is potent and expensive. Consider everyone with a positive test result. How many of them actually have the disease? Using the probabilities given, we see that the probability of having a positive result is 2.96% whereas the probability of having a positive result *and* having the disease is 0.98% – so two-thirds of the people who test positive do not have the disease and do not need the drug administering to them. This would be considered to be unacceptable.

A similar scenario of false negatives and positives can be applied when looking at errors from biometric readings, for example when logging on to a computer using fingerprint technology or, more disturbingly, at an international airport checking biometric readings against security databases. False positive readings can lead to a headache for those involved, whilst false negatives could allow real criminals to slip through the net.

The statistics we use offer the chance to refine and improve upon processes that impact on our daily lives in ways we shouldn't take for granted.

what's the point of...

QUADRATIC EQUATIONS?

The Beautiful Game? Oh ballistics...

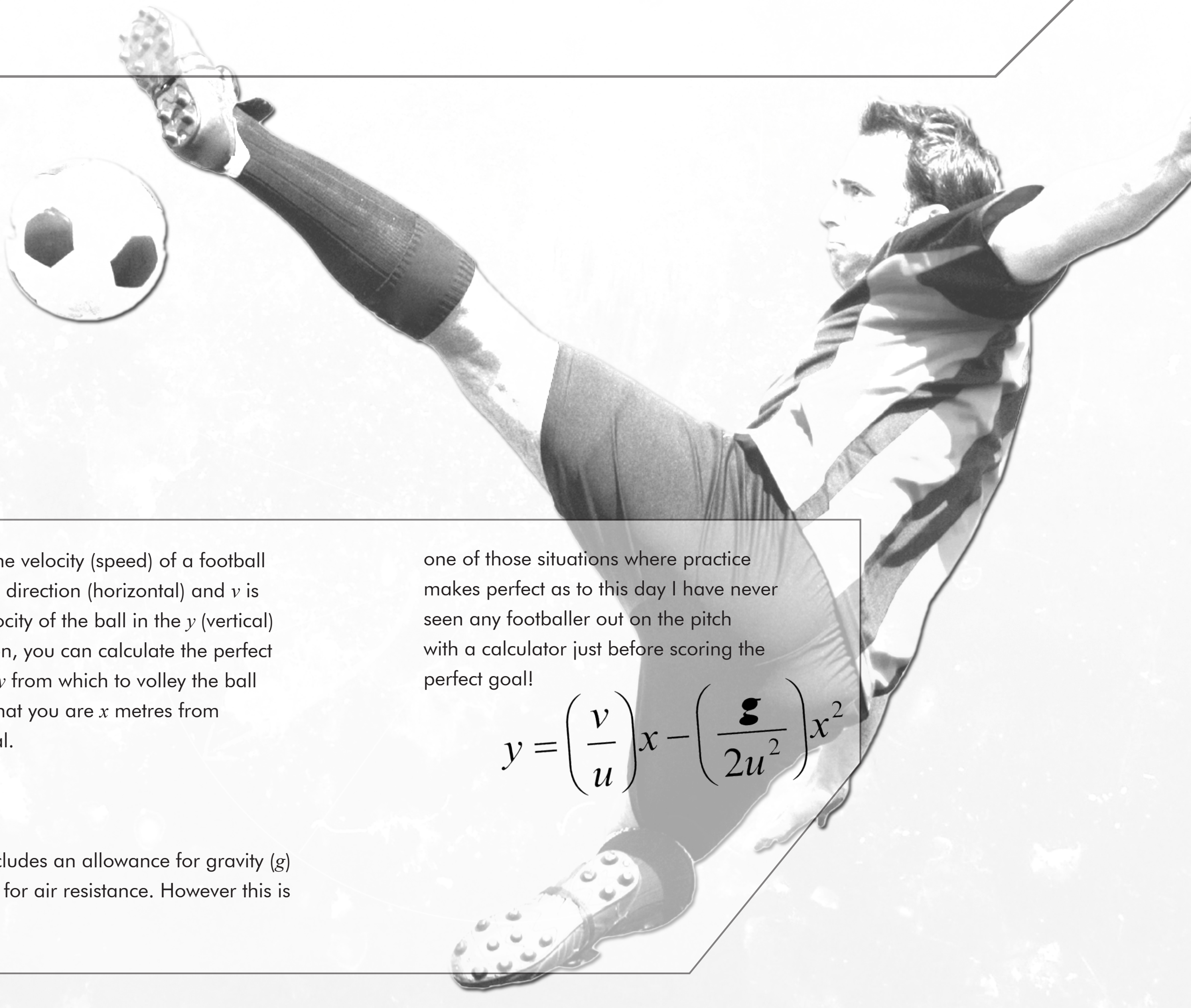
There will always be debate about issues in football. Who scored the best goal? The best ever player? The best team?

These debates will take up many hours and there will never be an outright winner, although Brazil and Liverpool are my choices – and I'm always right!

However there is no debate about one of the most technically gifted players of the modern era, Zinedine Zidane, who scored arguably the best ever goal in 2002 in the UEFA Champions League Final.

How did he do it?

Well, quadratic equations may help to explain the art of the volley. The principles are based on discoveries by Galileo and have many implications for military and sports enthusiasts alike.



If u is the velocity (speed) of a football in the x direction (horizontal) and v is the velocity of the ball in the y (vertical) direction, you can calculate the perfect height y from which to volley the ball given that you are x metres from the goal.

This includes an allowance for gravity (g) but not for air resistance. However this is one of those situations where practice makes perfect as to this day I have never seen any footballer out on the pitch with a calculator just before scoring the perfect goal!

$$y = \left(\frac{v}{u}\right)x - \left(\frac{g}{2u^2}\right)x^2$$

- Written and edited by Zia Rahman and Vivien Easson, More Maths Grads, School of Mathematical Sciences, Queen Mary, University of London (QMUL)
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Crime Scene Mathematics

Quadratic equations have also been applied to the saving of lives and the analysis of crime scenes.

When a forensics team reaches a crime scene where bullets have been fired, the application of quadratics helps to determine where a bullet was fired from.

When investigators arrive at the scene of a car crash they can work out the speed of the car at the time of the accident and make judgements on dangerous driving, etc. A car can travel from A to B by travelling at a constant speed. However in order to reach that speed it must accelerate and, using common sense, in order to stop it must decelerate (braking).

Where s is the distance travelled by a car, u is the velocity of the car, a is the acceleration and t is the time, we have a quadratic equation that links s to t.

$$s = ut + \frac{1}{2}at^2$$

If we substitute a negative value for a, then we can model deceleration and hence braking distance s. This simple equation predicts that by doubling your speed it will quadruple your stopping distance.

$$s = \frac{u^2}{2\mathrm{a}}$$

It makes sense to drive safely – and the maths proves it …

Raindrops keep falling on my head... but satellites don't

Let's perform a simple experiment. You throw a tennis ball in the air and (hopefully without hitting anyone) it should come back down having followed a parabolic path after obeying the laws of gravity. This path is essentially a quadratic equation with a negative coefficient for x^2 (why?).

In this age of rapid technological advances, we are continually and increasingly reliant on satellite technology. Without satellites there wouldn't be international mobile phone conversations, access to thousands of media channels, personal navigation systems, weather monitoring, etc. So why aren't satellites falling on our heads like the tennis ball? Think about a satellite being launched. Let's assume the Earth is stationary and completely flat along the x axis. At some point the satellite will fall back down to Earth, and this would be the range of the satellite. However the Earth is spherical, not flat, so the position of the x axis changes as we move around the Earth.

To try to understand this, draw a series of regular polygons by increasing the number of sides, n, each time by one (triangle, square, pentagon, hexagon, etc.). The more sides to the shape, the more it resembles a circle. In fact, consider a polygon with infinite sides. What shape is this? Each of the sides can be thought of as being the x axis but from a different point along the Earth's surface.

Every time the satellite reaches its range (where you expect it to land if the Earth had a flat surface), it actually hasn't. It will miss the edge because the Earth has a curved surface and so it has new x axis position. Furthermore, with the Earth actually rotating, a satellite can be launched to a precise height and speed to maintain geostationary orbit, appearing as if it were stationary above the same point on the Earth's surface, whilst actually keeping pace with the Earth's rotation. If we didn't have this, we would keep losing satellite TV feeds and end up watching less TV.

Now there's a thought …

what's the point of...

SEQUENCES?

Counting the cost or splashing out?

You struck it lucky and won £5000 in a prize draw. Having spent some of the cash on a new jacket and festival tickets, you decide to put £4000 of the money in a savings account. But which bank? And what does 5% AER mean?

Your friend wasn't so lucky and is in debt. She owes £300 on her store card and wants to know how fast she needs to pay it off. Being able to work with sequences of numbers is vital for anyone working in the financial sector.

AER stands for the annual equivalent rate and is the percentage of your £4000 that you'll get in interest at the end of the year.

At 5% AER you'll get £200 interest after twelve months. If you've not spent any of those savings then after two years you'll have 5% of £4200, or £210 more interest. The sequence £4000, £4200, £4410, £4630.50, … is calculated by taking each amount in turn and multiplying it by 105%, or 1.05. After ten years of saving you'll have £6515.58.

How could you make more money? Use your maths skills to get a job working for the bank!

See Facts and Figures below for details of salaries you could earn using your maths skills in a bank.

Your friend can use mathematical functions found on a spreadsheet computer package like Microsoft Excel to work out what she should pay each month.

Shopping shopping shopping!

In most jobs a computer can do the boring bits of the calculations but you would be expected to know enough about how it works to check it's giving a good answer or to explain it to a client or colleague.

Facts & Figures

In 2007 the average graduate starting salary in the UK was £23 000.

The average salary for employed people aged between 22 and 29 years was £18 000 – £19 000.

25% of employed people aged between 30 and 39 years had a salary of less than £14 500.

Graduate starting salaries in investment banks averaged at around £36 000. They hire people with 2:1s in a numerate degree like maths or science. They may also look at candidates' school performance.

Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
Work out what the rule is.
What number comes next?
If you look at the ratio of one term to the previous term, this value tends to the golden ratio: $\frac{1 + \sqrt{\ }}{2} = 1.6180339...$

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Compression compression compression

How do you listen to music? How did your parents?

Thirty years ago if you wanted to listen to music you had to carry around a large and heavy radio. Walkmans, the first personal stereo players, were just coming in.

These days your iPod can fit easily in your pocket. It has more computing power than existed in the world in 1950.
How do they get it so small?

Better computer memory can now hold far more data than before. Maths helps microchip designers to make microchips smaller and smaller each year.

But there's more to iPods than just the memory chips inside. A music file which takes up 10 MB of memory when stored on your hard disc can be compressed to a 1 MB file which fits better on your iPod. How does this work?

In the 1930s the American mathematician Claude Shannon invented a new science called information theory. We can understand the text message "c u l8r" even though letters are missing from all of the words. Some of the letters are redundant, and some of the letters contain the information. Redundancy is taken out in the process of compression to make files smaller. This is why the mp3 files played by an iPod are smaller: they've been compressed.

What's this got to do with sequences? Well, a sound wave can be written as a sum of different sine waves, and compression is a process that works with the sequence of these sine waves. This maths is called Fourier analysis and was invented over 200 years ago in France to investigate heat waves. Fourier analysis is widely used in many fields of science and engineering.

Power dressing

Stylists and designers work with shapes, colours and materials to create new fashions and update styles. Computer animation designers also need to create images, but they have to write them in mathematical language.

A cornrow braid hairstyle depends on a geometric sequence. Geometric means that each term in the given sequence is the same multiple of the previous term. So for example 1, $^1/_2$, $^1/_4$, $^1/_8$, $^1/_{16}$, … is a geometric sequence where each number is half of the previous one. You can see the braid getting smaller like this as it curls in on itself. In order to make the style fit the person, a hairdresser has to judge how much hair they use in each bit of the braid.

Hair stylists use their experience to make a hairstyle look good rather than writing down the maths. But what if you were playing a computer game where your character's hair has to move realistically? Lara Croft's ponytail swings perfectly in Tomb Raider because it's generated by a mathematical sequence. Someone's figured out the right equations to make it look real!

Cornrow braid hairstyle

Websites to check out:

www.mathscareers.org.uk
plus.maths.org

Interview with two designers with a maths/science background: plus.maths.org/issue39/interview/index.html

Interview with an accountant who studied maths and PE: plus.maths.org/issue2/career/index.html

what's the point of...

TRIGONOMETRY?

You can run... but you can't hide (forever...)

Going back to early July 2005, London was alive. But no sooner than the announcement for the 2012 Olympics been made, than Londoners were caught unawares by devastating acts of terrorism. The world is not always a safe place - but a little maths can help to make it a lot safer.

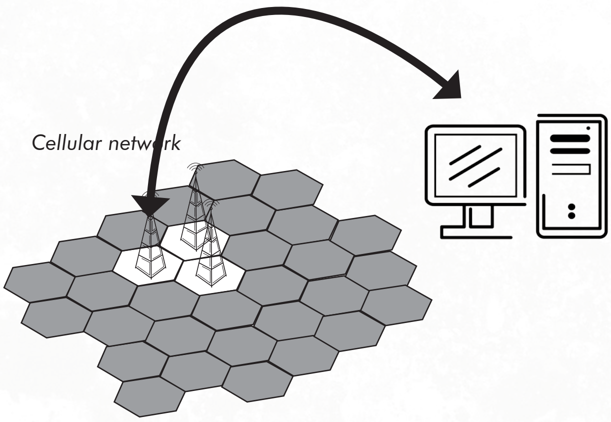
With security services across the world on full alert, the hunt was on for those responsible for the failed attacks of July 21st.

One of the suspects had fled to Rome in Italy, and took his mobile phone having changed his SIM card in the process. However a mobile phone can be tracked in two ways - using a unique identifier sent by the SIM card, and also by using a unique identifier sent by the handset (IMEI number).

Distances and angles between transmitters on a mobile phone network can help track phone users using:

$$\frac{A}{\sin A} = \frac{B}{\sin B} = \frac{C}{\sin C}$$

Cellular network

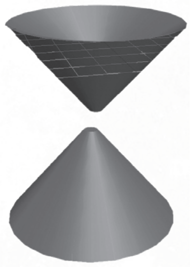


Using transmitters which are positioned at known locations, the minute a call is made from a handset, it is relatively simple to work out the location of the user using the sine rule, as their location is often the third point in a triangle.

Geometry and trigonometry also have huge roles in civil and military applications including locating aircraft through multilateration and hyperboloid shapes. This is based on the following principle: If a signal is sent from one location then receivers in different locations will get those signals but at different times. This is very useful for tracking aircraft and satellites.

Is Big Brother really watching you or is the world a safer place for all the surveillance? There are some questions maths can't answer...

Multilateration



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\frac{A}{\sin A} = \frac{B}{\sin B} = \frac{C}{\sin C}$$

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Welcome to Hollywood

Have you ever watched an animated movie, and thought 'how do they do that?' The chances are it is not just tracing paper and colouring pens…

The maths learnt at GCSE and A Level can actually help bring animated movies to life.

Tony DeRose is a computer scientist at Pixar Animation Studios. He realised his love of mathematics could transfer into the real world and a really interesting job by bringing the pretend world of animation to life. "Without mathematics we wouldn't have these visually rich environments and visually rich characters," explains Tony.

Advances in maths can lead to advances in animation. Earlier maths techniques show simple, hard, plastic toys. Now, advances in maths help make more human-like characters and special effects. DeRose explains the difference a few years can make, "You didn't see any water in *Toy Story*, whereas by the time we got to *Finding Nemo*, we had the computer techniques that were needed to create all the splash effects." How do maths classes help with the animation?

Trigonometry helps rotate and move characters, algebra creates the special effects that make images shine and sparkle and calculus helps light up a scene.

DeRose encourages people to stick with their maths classes. He says, "I remember as a mathematics student thinking, 'Well, where am I ever going to use simultaneous equations?' And I find myself using them every day, all the time now."

Even simple triangles rotating in 3D can produce results that are winning Oscars, including the manipulation of Gollum from *Lord of the Rings*.

From modern art to computer games to architecture – the humble triangle has come a long way from the text books of the ancients…

Where will your maths skills take *you*?

Need a job? Know your trig!

The rough with the smooth, good times and bad times, highs and lows. There are many clichés that describe the phenomenon of the boom-slump cycle.

Did you know that using trigonometry we can forecast when there are going to be bad times and when there are going to be good times in the economy? Financial analysts and politicians use this knowledge to plan for times of high unemployment and for making investment decisions. The peaks represent times of high employment and the troughs represent times of high unemployment.

Maths helps in planning your future. Can you plan a future without maths?